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BACKSCATTERING OF RADAR WAVES BY VEGETATED TERRAIN.(U)

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BACKSCATTERING OF RADAR WAVES BY VEGETATED TERRAIN

by
Richard A. Hevenor

June 1977

U.S. ARMY ENGINEER
TOPOGRAPHIC LABORATORIES
FORT BELVOIR, VA 22060

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This report presents a vector theory for the backscattering of electromagnetic radar waves from vegetation. The basic technique employed in the solution required simulating the vegetation with a random medium. This medium possesses an electrical permittivity that is generated by a continuous random process and is characterized by a particular probability density function. A solution for the radar backscatter coefficient is obtained in terms of the statistical characteristics of the random medium. A comparison of the theory with experimental data is given.

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Insight is given into the nature of depolarization, but explicit results for the depolarized terms are not obtained at this time because of the complexity and difficulty of the solution. Some of the conclusions of this work are

1. A theory has been developed for computing the like polarized (HH and VV) radar backscatter coefficients from certain types of vegetation by using a vector renormalization approach.

2. No rigorous quantitative comparison of theory with experiment was possible; however, qualitative comparisons indicate reasonable agreement. and

3. Although no explicit solution was obtained for the depolarization components, it was learned that one cause of depolarization is the anisotropy associated with the correlation function of the dielectric fluctuations.

PREFACE

The authority for performing the work described in this report is contained in Project 4A161102B52C, Research in Geodetic, Cartographic, and Geographic Sciences (Analysis of Radar Backscatter from Terrain).

The theory described is the result of an in-house study and is an application of the work of others in the field of electromagnetic wave scattering from random media. The result is a mathematical model for calculating the radar backscatter from certain types of vegetation. The technique used in the solution is the renormalization formulation. The author wishes to thank Professor Roger H. Lang of the George Washington University, Dr. Adrian K. Fung of the University of Kansas, and Dr. Eugene Margerum of the U.S. Army Engineer Topographic Laboratories (USAETL) for assistance in reviewing and developing the theory described herein. This task was performed under the supervision of Mr. Bernard B. Scheps, Chief, Technology Development Branch, Mr. Alphonse Elser, Chief, Geographic Information Systems Division, and Dr. Kenneth R. Kothe, Director, Geographic Sciences Laboratory. The work was under the general direction of Mr. Robert P. Macchia, Technical Director, U.S. Army Engineer Topographic Laboratories.

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INTRODUCTION

Purpose. This report presents a vector theory for the backscattering of electromagnetic radar waves from vegetation. The basic technique employed is that of simulating the vegetation with a random medium and then calculating the scattered electromagnetic field from the random medium.

Background. At the U.S. Army Engineer Topographic Laboratories, work on the problem of radar backscattering from natural terrain surfaces has been in progress for many years. However, this work differs from previous work because it attempts to model backscatter from realistic, extended targets (terrain) rather than simple geometric objects or surfaces. After the problem was defined and a literature review was completed, the first work calculated the radar backscatter from a bare ground surface with only large undulations. After this, the complexity of the bare ground problem was increased by adding on small undulations.[1] Next, the problem of scattering from a slightly rough surface with a lossy layer was considered.[2] However, the mathematical models developed for these situations are not applicable to radar scattering from vegetation or cultural features. It was decided that wave scattering from vegetation presented the most formidable problem and therefore should be attacked first. This report presents in detail the analysis performed in obtaining an approximate mathematical model for radar scattering from vegetation. The work included: (1) a brief survey of the available literature; (2) the development of a model for vegetation; (3) the derivation of equations for the radar backscatter coefficients for the model developed; (4) an analysis of parameter variations; and (5) the comparison of the developed theory with some experimental data. Throughout the study, the X band (8 GHz-12 GHz) frequency range was emphasized; however, the developed solutions are also applicable to certain other frequency regions, such as K, and Ku bands.

There are two direct applications for the results of this work. The first application relates to the military geographic analysis problem. It is hoped that by relating radar backscatter to an appropriate vegetation model, one will be able to quantitatively analyze and/or classify agricultural crops, forests, marshes, and other vegetation from radar returns; also it may be possible to compute vegetation parameters such as height, density and moisture from radar returns. The knowledge of such parameters could be

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1 R. A. Hevenor, "Backscattering of Electromagnetic Waves from a Surface Composed of Two Types of Surface Roughness," U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, Technical Report ETL-TR-71-4, October 1971, AD 737 675.

2 R. A. Hevenor, "Backscattering of Electromagnetic Waves from a Slightly Rough Surface with a Lossy Layer," U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, Technical Report ETL-TR-74-10, December 1974, AD A013 863.

important in the military analysis of terrain. Also, a quantitative model can be used to help develop radar requirements for military geographic applications. The second application relates to the development of radar image simulations. At present, most radar image simulations develop graytone values based upon a limited qualitative (subjective) understanding of the nature of radar returns. If one had a quantitative theory to explain the nature of radar returns that compared reasonably well with experiment, then the graytone values could be assigned more objectively. Also, fundamental understanding of radar scattering from vegetation will probably lead to other gains not yet contemplated.

Before proceeding to the development of a vegetation model, a brief survey of past contributions to this problem will be given. Consideration will first be given to both high and low frequency regions. The low frequency region will consist of the VHF, UHF, and L bands; and the high frequency regions will consist of the X, Ku, K, and Ka bands. The low frequency region was studied for both communication and remote sensing purposes. Many researchers have examined wave propagation in jungles. Among these, Pounds and La Grone[3] found that in the VHF region (less than 200 MHz) they could replace the jungle by a lossy dielectric slab having the average dielectric constant of the jungle. Hagn and Parker[4] attempted to measure the average dielectric properties of a forest, and Tamir[5] and Tamir and Dence[6] showed that the lateral wave played an important part in point to point communication in the jungle for this frequency range. In addition, an extensive measurements program was carried out by the Atlantic Research Corporation[7] in the jungles of Thailand. The results of those studies and others are summarized in the proceedings of the "*Workshop on Radio Systems in Forested and/or Vegetated Environments.*"[8] In the L band region, the lossy dielectric slab method breaks down because of increased scattering effects. In this region, Rosenbaum and Bowles[9] modeled a forest with a random medium and computed the backscattering using a single scattering theory. In addition, Du[10] com-

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[3] D. J. Pounds, and A. H. La Grone, *Considering Forest Vegetation as an Imperfect Dielectric Slab,* Electrical Engineering Research Laboratory, University of Texas, Austin, Report 6-56, May 1963.

[4] G. H. Hagn, and H. W. Parker, *Feasibility Study on the Use of Open-Wire Transmission Lines, Capacitors and Cavities to Measure the Electrical Properties of Vegetation,* Stanford Research Institute, Special Technical Report 13, August 1966.

[5] Theodore Tamir, "On Radio Wave Propagation in Forest Environments," *IEEE Transactions on Antennas and Propagation,* Vol. AP-15, No. 6, November 1967.

[6] D. Dence, and T. Tamir, "Radio Loss of Lateral Waves in Forest Environments," *Radio Science,* No. 4, 1969.

[7] Jansky and Bailey, *Tropical Propagation Research,* Final Report, Volume 1, Engineering Department of Atlantic Research Corporation, Alexandria, VA, AD660318.

[8] J. R. Wait, R. H. Ott, and T. Telfer, (Editors), *Workshop on Radio Systems in Forested and/or Vegetated Environments,* Technical Report No. ACC-ACO-1-74, February 1974.

[9] S. Rosenbaum, and L. Bowles, "Clutter Return from Vegetated Areas," *IEEE Transactions on Antennas and Propagation,* Vol. AP-22, No. 2, March 1974.

[10] Li-Jen Du, *Rayleigh Scattering from Leaves,* Scientific Report No. 1, The Ohio State University Electro Science Laboratory, 21 January 1969.

puted the Rayleigh scattering from a forest of leaves of arbitrary orientation. An examination of the literature in the high frequency region shows that very little theoretical work has been done. At these higher frequencies, the effects of scattering become very important. In fact, it can be shown that the single scattering theory of Rosenbaum and Bowles is no longer adequate since multiple scattering effects become important. Recently, Donn and Peake[11] attempted to model a forest in this frequency region. They took multiple scattering into account by reducing the thickness of the vegetation and then used a single scattering theory on the reduced vegetation layer. Radar backscatter measurements in the high frequency region have been made from bridges, truck booms, and aircraft. Peake,[12] Goodyear Aerospace,[13] Ulaby[14] and others have made measurements of backscatter coefficients as a function of incidence angle for agricultural crops, trees, marshes, grass, and other vegetation. A review of the various measurement programs is contained in a report by King and Moore.[15] An examination of the literature shows that backscatter measurements from forests have been made by several people. In many cases, however, the angular coverage was limited and depolarization information was not available. Also, the radar calibrations used in the various measurements programs are different and inconsistant, so that comparison of the results among investigators is difficult. In addition, the data was accompanied by very little quantitative ground truth information. This is particularly true with respect to electrical parameters, such as complex dielectric constants of the vegetation.

In the next few paragraphs, we will model a forest by employing a random medium, and we will discuss techniques to calculate the radar backscatter coefficient from this modeled forest.

Modeling of Vegetated Terrain. In order to calculate the radar backscatter coefficient from vegetated terrain, the electrical permeability, permittivity, and the conductivity of the vegetated volume must be specified. The permeability for most forests is constant throughout and can be taken to have the same permeability as the free space value, μ_o. The permittivity, on the other hand, varies a great deal, since the values for vegetation are quite different from that for free space. The manner in which the permittivity varies is so complicated that it would be a formidable task to try and obtain a deterministic spatial variation model. Such a deterministic calculation would

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[11] C. Donn, and W. Peake, "The Generalized Lommell-Seeliger Cross Section of a Foliage Environment," United States National Committee, International Scientific Radio Union, 1974 Spring Meeting, 10-13 June.

[12] W. Peake, R. L. Cosgriff, and R. C. Taylor, *Terrain Scattering Properties for Sensor System Design (Terrain Handbook II)*, The Ohio State University, May 1960.

[13] Goodyear Aerospace Corporation, *Radar Terrain Return Study*, Final Report 30 September 1959.

[14] Fawwaz Ulaby, *Radar Response to Vegetation*, University of Kansas, CRES Technical Report 177-42, September 1973.

[15] C. King, and R. K. Moore, *A Survey of Terrain Radar Backscatter Coefficient Measurement Programs*, University of Kansas, CRES Technical Report 243-2, December 1973.

require knowing the exact location of every leaf, twig, and tree trunk in a forest. Even if this were possible, the calculation of the scattered electromagnetic field from such a complicated dielectric variation would be an enormous undertaking. And then, the results would be valid only for the particular forest where the dielectric variation was measured. If radar backscatter predictions were wanted over some other forest, the entire complicated deterministic procedure would have to be repeated. In order to bypass the difficulties of the deterministic solution, we will consider the dielectric variations in a forest to be generated by a random process. The symbol to be used for the relative dielectric constant will be $\epsilon_r(\underline{r})$. The vector $\underline{r}$ indicates that the relative dielectric constant is some arbitrary function of three mutually perpendicular spatial coordinates. We will now replace the forest with a random medium in which $\epsilon_r(\underline{r})$ is generated by a random process having the same mean, ϵ_a, and standard deviation, ϵ_s, as the original forest. To be specific, we will let

$$\epsilon_r(\underline{r}) = \epsilon_a + \epsilon_s \mu(\underline{r}) \qquad (1)$$

where $\mu(\underline{r})$ is a stationary random process with zero mean and unit variance. A simple calculation shows that

$$< \epsilon_r(\underline{r}) > = \epsilon_a \qquad (2)$$

$$< (\epsilon_r(\underline{r}) - \epsilon_a)^2 > = \epsilon_s^2 \qquad (3)$$

where the brackets are used to indicate the process of taking a statistical average. In addition, it will be necessary to assume some correlation function, $B(\underline{r} - \underline{r}')$, for $\mu(\underline{r})$. The definition of the correlation function can be stated as follows:

$$< \mu(\underline{r})\, \mu(\underline{r}') > = B(\underline{r} - \underline{r}') \qquad (4)$$

The correlation function has the property that it approaches zero as $|\underline{r}-\underline{r}'|$ approaches infinity. The correlation distance (or correlation length) is the distance at which $B(\underline{r} - \underline{r}')$ decreases to e^{-1} times its maximum value. When the correlation function is direction dependent, then the correlation function is considered to be anisotropic. If the correlation function is independent of direction, that is it depends only on $|\underline{r}-\underline{r}'|$, then it is considered an isotropic correlation function. In general, the correlation length is a measure of the average size of the particles in the random medium.

The conductivity in vegetation will also vary from point to point. However, the variations in conductivity should not produce major effects at the frequencies of interest. The major effect of conductivity is its average value, σ_a, which helps attenuate the waves.

Before proceeding to discuss some of the techniques that can be used to calculate the backscatter coefficient, we shall compute some typical values of ϵ_a, σ_a, and ϵ_s for a deciduous forest, to determine the order of magnitude of these parameters. We will assume a forest made up of leaves alone. Each leaf will be considered to be a rectangular solid in shape, with the dimensions of w x w x t. The dimensions of the entire forest will be L x L x D. The spacing between leaves in the horizontal direction is d; while the spacing in the vertical direction is a. The relative mean complex dielectric, $\hat{\epsilon}_a$, can then be computed as

$$\hat{\epsilon}_a = \frac{V_\ell N_\ell \hat{\epsilon}_\ell + V_A \epsilon_A}{V_T} = \epsilon_a - j(\sigma_a/\omega \epsilon_o) \quad (5)$$

where

V_ℓ = volume of one leaf = w x w x t.
N_ℓ = total number of leaves in the forest.
$\hat{\epsilon}_\ell$ = relative complex dielectric constant of one leaf.
V_A = volume of the forest which is solely air.
ϵ_A = relative dielectric constant of air = 1.0
V_T = the total volume of the forest = L x L x D.
ω = radian frequency.
ϵ_o = permittivity of free space.

Finding the dielectric constant of leaves of any type is difficult to do, since so little data has been obtained. One of the few reliable sources available was used, i.e. Broadhurst[16] who measured the complex permittivity of tulip tree leaves and branches for a broad range of frequencies. From his data, we find that in the X-band region, the relative complex permittivity of tulip tree leaves is approximately 40-j10. Using this value along with the following parameters, we can compute $\hat{\epsilon}_a$.

Let

L = 15 meters
D = 3 meters
w = 4 centimeters
t = 0.01 centimeter
d = 4 centimeters
a = 0.3 meter
$\hat{\epsilon}_\ell$ = 40-j10 = $\epsilon_\ell' - j\epsilon_\ell''$
ϵ_A = 1.0

[16] M. G. Broadhurst, *Complex Dielectric Constant and Dissipation Factor of Foliage*, National Bureau of Standards, Report 9592, 1970.

The above set of numbers will allow for the calculation of N_ℓ, V_ℓ, V_A, and V_T. When these parameters are calculated and placed in the above equations, we obtain for $\hat{\epsilon}_a$

$$\hat{\epsilon}_a = 1.003 - j\,0.0008$$

In a similar manner, the variance, ϵ_s^2, of the real portion of the complex permittivity of the forest of leaves can be computed by using the following equation:

$$\epsilon_s^2 = \frac{V_\ell \, N_\ell \, (\epsilon_\ell' - \epsilon_a)^2 + V_A \, (\epsilon_A - \epsilon_a)^2}{V_T} \tag{6}$$

If the data given above is used, ϵ_s^2 can be easily calculated to give the following value:

$$\epsilon_s^2 = 0.126$$

The calculated value of ϵ_a agrees with the open wire transmission line results presented by Hagn and Parker.[17] It can be seen that even though ϵ_ℓ' is very large, the average dielectric is approximately one because of the small percentage of the total volume that the leaves occupy.

A forested terrain will be modeled by a flat earth covered by a slab of random medium. The random medium is described by its dielectric variation given by equation (1). A plane wave from free space is assumed to be incident upon the slab at angle θ_i with respect to the normal. The objective then is to calculate the backscatter coefficients for horizontal and vertical polarizations. This model of the forest can be simplified further when the frequencies of interest are X-band and above. Because of the average loss due to water content and losses due to multiple scattering, the incident wave will be completely absorbed in most cases before reaching the ground. As a result, we can neglect the effect of the ground and assume that the backscatter comes solely from the forest medium. This is equivalent then to solving the problem of a plane wave incident upon a half space of lossy random media. A number of techniques will be briefly discussed that could be used to calculate the backscatter from a lossy random half space.

Single Scattering – Born Approximation. This technique was one of the first to be used for the calculation of scattering from random media. Basically, the method entails dividing the scattering medium into a large number of individual scattering elements. The scattering from each element is computed separately and independently from all other elements. Then, the total scattered field is obtained by adding the fields due to each individual scattering element. Mathematically, the

[17] G. H. Hagn and H. W. Parker, *Feasibility Study on the Use of Open-Wire Transmission Lines, Capacitors and Cavities to Measure the Electrical Properties of Vegetation*, Stanford Research Institute, Special Technical Report 13 August 1966.

problem involves an integral equation for the scattered field, in terms of the total field (incident field plus scattered field). The scattered field within the integral is set equal to zero in order to obtain a first approximation. Then the incident wave acts like a source for the scattered wave. When the scattering medium is lossy, those elements far enough from the surface see an incident field that is highly attenuated. As a result, their contribution to the backscattered field is small. For this reason, the vegetation layer appears to have an equivalent thickness from which most of the scattering occurs. Since this technique does not consider multiple scattering effects and is limited to media that are slightly random, that is $\epsilon_s \ll 1$, it was decided not to explore this technique any further.

Multiple Scattering. In the single scattering method just discussed, the random medium was broken up into many mutually independent scatterers. The assumption of mutual independence among scatterers is in general not valid, and one must consider the coupling between scatterers. This mutual coupling can be accounted for by including in the backscattered field contributions from waves that have been scattered more than once. There presently exists many methods that can be used for calculating the effect of multiple scattering. The accuracy of each method and its region of validity are questions of current research and have not been completely resolved to date. A brief outline of each method will be given in the following paragraphs.

Scalar Renormalization. The total field in the random medium is set equal to the sum of a coherent mean wave and a scattered wave. Two equations are derived from the scalar wave equation that allow for approximate solutions of the mean wave and the scattered wave. These two equations are referred to as the renormalized equations for the mean and scattered waves. The solution for the scattered wave involves the mean wave, and so the mean wave must be calculated first. In fact, the mean wave acts like a source term that generates the scattered wave. The solution of the equations for the mean wave will lead to a dispersion equation. This dispersion equation, when solved, will yield an effective dielectric constant for the mean wave. This effective dielectric constant will be complex even if the mean conductivity in the random medium is zero. The interpretation that has been placed on this is that the imaginary part of the effective dielectric constant arises from multiple scattering effects. Thus, multiple scattering is considered when the scattered wave is calculated. The unknown factor in this technique is how much multiple scattering is really considered. Also, the scalar approach cannot allow polarization effects to be considered. In order to consider polarization effects, one would have to use a vector wave equation and be certain that the real causes of depolarization are included in the solution.

Vector Renormalization. The approach taken here is similar to the scalar renormalization approach, except that now a vector wave equation is considered and

depolarization terms can be obtained. As with the scalar approach, a dispersion equation is obtained, and an effective dielectric constant is found. Now, however, the effective dielectric constant turns out to be a 3 by 3 matrix (tensor); whereas in the scalar case, it is a scalar. Basically, this points out that we are replacing the random medium by a deterministic anisotropic medium. The elements of the effective dielectric tensor are found to involve the correlation function of the random medium. The mean wave is then computed, as in the scalar case, by using the solution from the dispersion equation. The scattered field can then be calculated by using the solution for the mean wave. This technique was the one chosen for thorough analysis, and it is the main subject of the rest of this report.

Other Techniques. A number of other techniques exist that could be applied to the vegetation problem. One such technique is the diffusion method. This method involves deriving an equation for the probability density function of the field's amplitude and phase. Once an approximate solution to this equation has been obtained, the radar backscatter coefficients can be readily computed. This technique has been successfully carried out in the one dimensional case,[18] i.e., the case where it is assumed that the dielectric constant varies only in one direction.

Two other techniques that could be used are the radiative transfer method and the discrete scatterer technique. The radiative transfer technique is a phenomenological theory that attempts to calculate intensity using the assumption that the power radiated from individual scattering regions adds incoherently. Alternatively, the discrete scatterer technique makes use of the far field radiation characteristics of individual scatterers to obtain backscatter characteristics for a collection of these scatterers.

Another technique is computer simulation in which the scattered power would be computed by numerical methods for many different sample values of the random media. The mean value of the scattered power for the ensemble could then be computed from the individual sample values.

ANALYSIS

The vector renormalization approach chosen for analysis in this report requires defining the geometry of the scattering problem and the approximate solution of wave propagation in a random medium of infinite extent in order to derive equations for the calculation of horizontal and vertical polarization backscatter coefficients. Equations (129) and (175) are the final results of all the derivations in the following paragraphs. Consider the problem of a plane electromagnetic wave incident obliquely at an angle θ_i from a free space medium ($z > 0$) onto a lossy random medium ($z < 0$). The geome-

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[18] R. H. Lang, "Probability Density Function and Moments of the Field Inside a One-Dimensional Random Medium," *Journal of Mathematical Physics*, Vol. 11, No. 12, December 1973.

try of the problem is shown in figure 1.

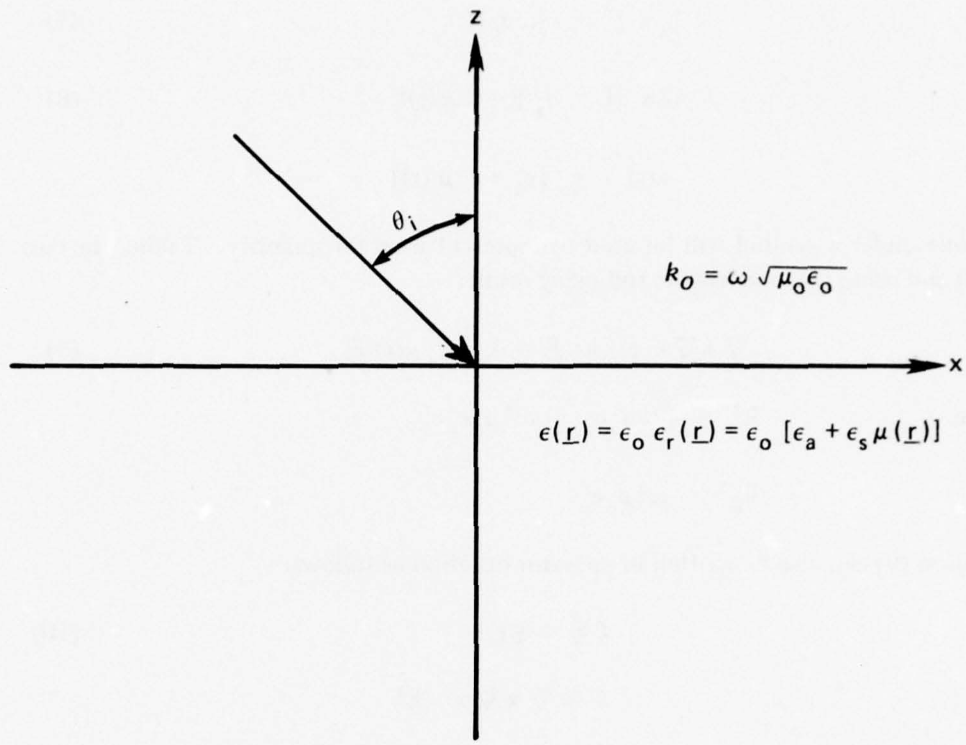

Figure 1. Scattering Geometry

The lossy random medium is characterized by a dielectric, $\epsilon(\underline{r})$, which is composed of the sum of a random part and a nonrandom part. The nonrandom part, $\epsilon_o \epsilon_a$, is equal to the mean of $\epsilon(\underline{r})$. The random portion comes from $\mu(\underline{r})$, which is generated by a stationary random process with zero mean and a variance equal to one. The objective is now to obtain an approximate expression for the backscattered far field in the upper medium. This field can then be used to compute the radar backscatter coefficients. In order to compute the scattered field in the upper medium, one has to first obtain an understanding of the nature of wave propagation and scattering in a random medium of infinite extent. This will be done using the renormalization formulation. The method of renormalization was developed in detail by Tatarskii and Gertsenshtein.[19]

The electric ($\underline{E}$) and magnetic ($\underline{H}$) fields in a random medium can be related by Maxwell's equations in time harmonic form. The time harmonic variation will be

[19] V. I. Tatarskii, and M. E. Gertsenshtein, "Propagation of Waves in a Medium with Strong Fluctuations of the Refractive Index," *Soviet Physics JETP*, Vol. 17, No. 2, August 1963.

exp (jωt). We will assume a rectangular Cartesian coordinate system to be imbedded in the random medium of infinite extent.

$$\nabla \times \underline{E} = -j\omega\mu_o \underline{H} \tag{7}$$

$$\nabla \times \underline{H} = \sigma_a \underline{E} + j\omega\epsilon(\underline{r})\underline{E} \tag{8}$$

$$\epsilon(\underline{r}) = \epsilon_o \left[\epsilon_a + \epsilon_s \mu(\underline{r})\right]$$

The line under a symbol will be used to represent a vector quantity. Taking the curl of (7) and using (8), one has the following result:

$$\nabla \times \nabla \times \underline{E} - k^2 \underline{E} = k_o^2 \epsilon_s \mu(\underline{r}) \underline{E} \tag{9}$$

where $$k^2 = -j\omega\mu_o\sigma_a + \omega^2 \mu_o \epsilon_o \epsilon_a$$

$$k_o^2 = \omega^2 \mu_o \epsilon_o$$

Equation (9) can also be written in operator notation as follows:

$$\mathcal{L} \underline{E} = \xi \underline{E} \tag{10}$$

$$\mathcal{L} \equiv \nabla \times \nabla \times - k^2$$

$$\xi = k_o^2 \epsilon_s \mu(r)$$

We will now assume that the total electric field ($\underline{E}$) in the random medium can be written as the sum of a mean wave and a scattered wave.

$$\underline{E} = \langle \underline{E} \rangle + \underline{E}_s \tag{11}$$

$\langle \underline{E} \rangle$ is the mean wave

$\underline{E}_s$ is the scattered wave

Obviously the mean of $\underline{E}_s$ is zero. However, the mean of $\underline{E}_s \cdot \underline{E}_s^*$ will not be zero. The star indicates taking the complex conjugate. Placing equation (11) into (10) and taking the average of the resultant expression will give

$$\mathcal{L} \langle \underline{E} \rangle = \langle \xi \underline{E}_s \rangle \text{ since } \langle \xi \rangle = 0 \tag{12}$$

When (12) is subtracted from (10) we have

$$\mathcal{L}\,[\underline{E} - <\underline{E}>] = \xi\,\underline{E} - <\xi\,\underline{E}_s> \tag{13}$$

$$\mathcal{L}\,\underline{E}_s = \xi <\underline{E}> + \xi\,\underline{E}_s - <\xi\,\underline{E}_s> \tag{14}$$

$$\underline{E}_s = \mathcal{L}^{-1}\,[\xi <\underline{E}>] + \mathcal{L}^{-1}\,[\xi\,\underline{E}_s - <\xi\,\underline{E}_s>] \tag{15}$$

The inverse operator $\mathcal{L}^{-1}$ can be written in terms of an infinite space dyadic Green's function.

$$\mathcal{L}^{-1}\;(\quad) = \int_{v'} (\quad)\cdot\underline{\underline{\Gamma}}\,(\underline{r},\underline{r}')\;\underline{dr}' \tag{16}$$

where $\underline{dr}' = dx'dy'dz'$

$\underline{\underline{\Gamma}}\,(\underline{r},\underline{r}')$ is the infinite space dyadic Green's function, where the double line under $\underline{\underline{\Gamma}}$ is used to indicate a dyadic. The form of $\underline{\underline{\Gamma}}\,(\underline{r},\underline{r}')$ is as follows:

$$\underline{\underline{\Gamma}}\,(\underline{r},\underline{r}') = \left[\underline{\underline{I}} - \frac{\nabla\nabla'}{k^2}\right]\frac{e^{-jk|\underline{r}-\underline{r}'|}}{4\pi\,|\underline{r}-\underline{r}'|} \tag{17}$$

where $\underline{\underline{I}}$ is the unit dyad. . A good derivation of the dyadic Green's function can be found in the monograph by Chen-To Tai.[20] The integration in (16) is to be carried out over the entire volume of the random medium, which in this case is over all space. When the equation for $\underline{E}_s$ as given by (15) is placed in equation (12), the following result is obtained:

$$[\mathcal{L} - <\xi\mathcal{L}^{-1}\xi>]\;<\underline{E}> = <\xi\mathcal{L}^{-1}\,[\xi\underline{E}_s - <\xi\underline{E}_s>]> \tag{18}$$

For a first approximation of the mean wave, we will consider the right hand side of (18) to be zero.

$$[\mathcal{L} - <\xi\mathcal{L}^{-1}\xi>]\,<\underline{E}> = 0 \tag{19}$$

An equation for the scattered wave in terms of the mean wave can be obtained by using (15). The second term on the right hand side of (15) will be considered small in comparison with the first term involving the mean wave.

$$\underline{E}_s = \mathcal{L}^{-1}\,[\xi<\underline{E}>] \tag{20}$$

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20 Chen-To-Tai, *Dyadic Green's Functions in Electromagnetic Theory*, International Textbook Company, 1971.

Equations (19) and (20) are the pair of renormalization equations. The first observation that should be made is that the mean wave does not propagate with the propagation constant k. One must first derive a dispersion equation from (19) and then solve *this equation for an effective propagation constant. This effective propagation constant* will describe the attenuation and phase characteristics of the mean wave. The second important observation that should be made is that the mean wave acts like a source for the scattered wave as shown by equation (20). When the quantities for $\mathcal{L}$, ξ, and $\mathcal{L}^{-1}$ are placed in (19), the following expression results:

$$\nabla \times \nabla \times \langle \underline{E}(r) \rangle - k^2 \langle \underline{E}(r) \rangle - k_o^4\, \epsilon_s^2 \int_{V'} \langle \mu(\underline{r})\mu(\underline{r}') \rangle \langle \underline{E}(\underline{r}') \rangle \cdot \underline{\Gamma}(r, \underline{r}')\, d\underline{r}' = 0 \tag{21}$$

The quantity $\langle \mu(\underline{r})\mu(\underline{r}') \rangle$ was defined earlier as the correlation function. Also, it should be noticed that within the volume integral the mean wave is a function of $\underline{r}'$; whereas the two terms involving the mean wave outside the integral are functions of $\underline{r}$. This is because everything to the right of the $\mathcal{L}^{-1}$ operator must be included in the integral. Also, it can be shown that because of the symmetrical properties of the dyadic Green's function the following equation can be written:

$$\langle \underline{E}(r') \rangle \cdot \underline{\underline{\Gamma}}(r, r') = \underline{\underline{\Gamma}}(r, r') \cdot \langle \underline{E}(r') \rangle \tag{22}$$

We will seek plane wave solutions to (21) which will have the following form:

$$\langle \underline{E}(\underline{r}) \rangle = \underline{A}\, e^{-j\underline{K}\cdot\underline{r}} \tag{23}$$

$\underline{A}$ is a constant vector

$\underline{K}$ is the effective propagation constant

The effective propagation constant $\underline{K}$ is complex and will be complex even if k is real ($\sigma_a = 0$). The complex portion of $\underline{K}$ will attenuate the mean wave. This attenuation has been interpreted as arising only from *multiple scattering effects, when σ_a is zero.* In general then, there are two factors that contribute to the attenuation of the mean wave. The first contribution is due to the average loss in the random medium that arises from a finite value of σ_a. The second contribution comes from the effects of multiple scattering. The mean wave that appears inside the volume integral can be written as

$$\langle \underline{E}(\underline{r}') \rangle = \underline{A}\, e^{-j\underline{K}\cdot\underline{r}'} \tag{24}$$

$$\langle \underline{E}(\underline{r}') \rangle = \underline{A}\, e^{-j\underline{K}\cdot\underline{r}}\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')} \tag{25}$$

$$\langle \underline{E}(\underline{r}') \rangle = \langle \underline{E}(\underline{r}) \rangle\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')} \tag{26}$$

With the aid of (22) and (26), equation (21) can be written as follows:

$$\nabla \times \nabla \times \langle \underline{E}(\underline{r}) \rangle - k^2 \langle \underline{E}(\underline{r}) \rangle - k_o^4 \, \epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \, \underline{\underline{\Gamma}}(\underline{r}-\underline{r}') \cdot \langle \underline{E}(\underline{r}) \rangle \, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')} \, d\underline{r}' = 0 \tag{27}$$

$$\left[\nabla \times \nabla \times - k^2 - k_o^4 \, \epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \, \underline{\underline{\Gamma}}(\underline{r}-\underline{r}') \, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')} \, d\underline{r}' \right] \langle E(r) \rangle = 0 \tag{28}$$

Now, let us consider an electromagnetic wave propagating in a deterministic anisotropic medium. The electric field for this wave will be designated by $\underline{E}_o$. The relative complex dielectric tensor will be $\hat{\underline{\underline{\epsilon}}}$. The vector wave equation for the electric field propagating in the anisotropic medium is as follows:

$$(\nabla \times \nabla \times - k_o^2 \, \hat{\underline{\underline{\epsilon}}}) \, \underline{E}_o = 0 \tag{29}$$

where in general $\hat{\underline{\underline{\epsilon}}}$ can be written as a 3x3 matrix.

$$\hat{\underline{\underline{\epsilon}}} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

When equation (28) is compared with equation (29), we see that they are of the same form and can be made equal if we allow the following two relations to hold:

$$\underline{E}_o = \langle \underline{E}(\underline{r}) \rangle$$

and

$$\hat{\underline{\underline{\epsilon}}} = \frac{k^2}{k_o^2} \, \underline{\underline{I}} + k_o^2 \, \epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \, \underline{\underline{\Gamma}}(\underline{r}-\underline{r}') \, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')} \, d\underline{r}'$$

Thus, it can be seen that the mean wave in a random medium can be considered as a wave propagating in a deterministic anisotropic medium. The relative complex dielectric tensor can be seen to be a function of the correlation function, $B(\underline{r} - \underline{r}')$, of the random medium. The dyadic Green's function can easily be written in rectangular component form so that the individual components of $\hat{\underline{\underline{\epsilon}}}$ can be determined.

$$\underline{\underline{\Gamma}}(\underline{r}-\underline{r}') = [\underline{a}_x\underline{a}_x + \underline{a}_y\underline{a}_y + \underline{a}_z\underline{a}_z]\, f_2(R) + \Bigg[\underline{a}_x\underline{a}_x \frac{(x-x')^2}{R^2} + \underline{a}_x\underline{a}_y \frac{(x-x')(y-y')}{R^2} +$$

$$\underline{a}_x\underline{a}_z \frac{(x-x')(z-z')}{R^2} + \underline{a}_y\underline{a}_x \frac{(y-y')(x-x')}{R^2} + \underline{a}_y\underline{a}_y \frac{(y-y')^2}{R^2} + \underline{a}_y\underline{a}_z \frac{(y-y')(z-z')}{R^2} +$$

$$\underline{a}_z\underline{a}_x \frac{(x-x')(z-z')}{R^2} + \underline{a}_z\underline{a}_y \frac{(z-z')(y-y')}{R^2} + \underline{a}_z\underline{a}_z \frac{(z-z')^2}{R^2}\Bigg]\, f_3(R)$$

Where

$\underline{a}_x$ is a unit vector in the x direction

$\underline{a}_y$ is a unit vector in the y direction

$\underline{a}_z$ is a unit vector in the z direction

$$R = |\underline{r}-\underline{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$f_2(R) = (k^2R^2 - jkR - 1)\, e^{-jkR} / (4\pi k^2 R^3)$$

$$f_3(R) = (3jkR + 3 - k^2R^2)\, e^{-jkR} / (4\pi k^2 R^3)$$

The individual elements of the relative complex dielectric tensor, $\hat{\underline{\underline{\epsilon}}}$, can now be written and are given below:

$$\epsilon_{xx} = k^2/k_o^2 + k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ f_2(R) + \frac{(x-x')^2}{R^2} f_3(R) \right\} e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{xy} = k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ \frac{(x-x')(y-y')}{R^2} \right\} f_3(R)\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{xz} = k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ \frac{(x-x')(z-z')}{R^2} \right\} f_3(R)\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{yx} = k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ \frac{(y-y')(x-x')}{R^2} \right\} f_3(R)\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{yy} = k^2/k_o^2 + k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ f_2(R) + \frac{(y-y')^2}{R^2} f_3(R) \right\} e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{yz} = k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ \frac{(y-y')(z-z')}{R^2} \right\} f_3(R)\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{zx} = k_o^2 \epsilon_s^2 \int_{v'} B(\underline{r} - \underline{r}') \left\{ \frac{(z-z')(x-x')}{R^2} \right\} f_3(R)\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{zy} = k_o^2 \epsilon_s^2 \int_{v'} B(\underline{r} - \underline{r}') \left\{ \frac{(z-z')(y-y')}{R^2} \right\} f_3(R)\, e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\epsilon_{zz} = k^2/k_o^2 + k_o^2 \epsilon_s^2 \int_{v'} B(\underline{r} - \underline{r}') \left\{ f_2(R) + \frac{(z-z')^2}{R^2} f_3(R) \right\} e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

It can easily be seen from the above equations that the dielectric tensor $\hat{\underline{\underline{\epsilon}}}$ is symmetric. Each of the elements of the dielectric tensor can be seen to be a function of $\underline{K}$. Therefore, not only the magnitude but also the direction of the effective propagation constant is important in determining each of the elements in the dielectric tensor. We will now attempt to obtain a dispersion equation from (29), which will allow a solution for $\underline{K}$. The operator $\nabla \times \nabla \times$ can be written in matrix form as given below:

$$\nabla \times \nabla \times = \begin{bmatrix} -\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2}\right) & \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial x \partial z} \\ \frac{\partial^2}{\partial x \partial y} & -\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right) & \frac{\partial^2}{\partial z \partial y} \\ \frac{\partial^2}{\partial x \partial z} & \frac{\partial^2}{\partial y \partial z} & -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \end{bmatrix}$$

By using the above definition and letting $\underline{E}_o = \langle \underline{E}(\underline{r}) \rangle = \underline{A} \exp(-j\underline{K} \cdot \underline{r})$, equation (29) becomes

$$\begin{bmatrix} K_z^2 + K_y^2 - k_o^2 \epsilon_{xx} & -K_x K_y - k_o^2 \epsilon_{xy} & -K_x K_z - k_o^2 \epsilon_{xz} \\ -K_x K_y - k_o^2 \epsilon_{yx} & K_z^2 + K_x^2 - k_o^2 \epsilon_{yy} & -K_y K_z - k_o^2 \epsilon_{yz} \\ -K_x K_z - k_o^2 \epsilon_{zx} & -K_y K_z - k_o^2 \epsilon_{zy} & K_y^2 + K_x^2 - k_o^2 \epsilon_{zz} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = 0 \qquad (30)$$

where K_x, K_y, and K_z are the components of $\underline{K}$ along the x, y, and z axes respectively. Likewise A_x, A_y, and A_z are the components of $\underline{A}$. It can be seen that the system of partial differential equations has been replaced by a system of linear algebraic equations. We will now switch from rectangular to spherical coordinates in our description

of the $\underline{K}$ vector. Figure 2 shows the definitions of θ and ϕ, the spherical angles that will be used to describe $\underline{K}$.

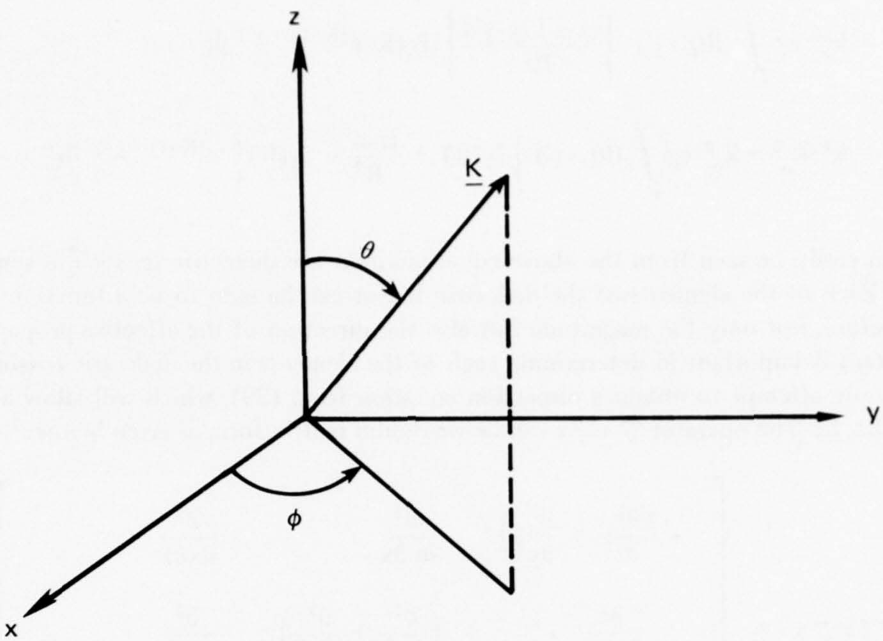

Figure 2. Geometry of the $\underline{K}$ Vector

The angle θ is the angle between the z axis and $\underline{K}$. The angle ϕ is the angle between the x axis and the projection of $\underline{K}$ onto the xy plane. The rectangular components of $\underline{K}$ can now be written as follows:

$$K_x = K \sin\theta \cos\phi$$

$$K_y = K \sin\theta \sin\phi$$

$$K_z = K \cos\theta$$

$$K = \sqrt{K_x^{\ 2} + K_y^{\ 2} + K_z^{\ 2}} = |\underline{K}|$$

The magnitude sign on $\underline{K}$ is meant to change a vector to a scalar and not to take the magnitude of a complex number. Thus, K can still be complex. We will now introduce the direction cosines of the $\underline{K}$ vector (α, β, γ), which can be determined from the equations above.

$$K_x = K\alpha$$

$$\alpha = \sin\theta\cos\phi$$

$$K_y = K\beta$$

$$\beta = \sin\theta\sin\phi$$

$$K_z = K\gamma$$

$$\gamma = \cos\theta$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Incorporating the above equations and definitions into equation (30) will yield the following:

$$\begin{bmatrix} K^2(\gamma^2+\beta^2) - k_o^2\,\epsilon_{xx} & -K^2\alpha\beta - k_o^2\,\epsilon_{xy} & -K^2\alpha\gamma - k_o^2\,\epsilon_{xz} \\ -K^2\alpha\beta - k_o^2\,\epsilon_{yx} & K^2(\gamma^2+\alpha^2) - k_o^2\,\epsilon_{yy} & -K^2\beta\gamma - k_o^2\,\epsilon_{yz} \\ -K^2\alpha\gamma - k_o^2\,\epsilon_{zx} & -K^2\beta\gamma - k_o^2\,\epsilon_{zy} & K^2(\beta^2+\alpha^2) - k_o^2\,\epsilon_{zz} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = 0 \quad (31)$$

In our first solution of (31), we will assume that all three equations are coupled. If this is true, then a nontrivial solution to (31) can be obtained only if the determinant of the square matrix is equal to zero. This will be satisfied if

$$K^4 - K^2\left(\frac{H}{D}\right) + \frac{k_o^6\,|\hat{\underline{\underline{\epsilon}}}|}{D} = 0 \qquad (32)$$

where H, D, and $|\hat{\underline{\underline{\epsilon}}}|$ are defined below:

$$\begin{aligned} H = k_o^4 \{ & (\gamma^2+\beta^2)\epsilon_{yy}\epsilon_{zz} + \epsilon_{xx}\,[\epsilon_{yy}(\beta^2+\alpha^2) + \epsilon_{zz}(\gamma^2+\alpha^2)] \\ & - \alpha\beta\,\epsilon_{yz}\epsilon_{zx} - \epsilon_{xy}\,[\epsilon_{yz}\,\alpha\gamma + \epsilon_{zx}\,\beta\gamma] - \alpha\gamma\,\epsilon_{zy}\epsilon_{yx} \\ & - \epsilon_{xz}[\alpha\beta\,\epsilon_{zy} + \epsilon_{yx}\beta\gamma] + \alpha\gamma\,\epsilon_{yy}\epsilon_{zx} - \epsilon_{xz}[\epsilon_{zx}(\gamma^2+\alpha^2) - \epsilon_{yy}\alpha\gamma] \\ & + \alpha\beta\,\epsilon_{yx}\epsilon_{zz} - \epsilon_{xy}\,[\epsilon_{yx}(\beta^2+\alpha^2) - \epsilon_{zz}\,\alpha\beta] - (\gamma^2+\beta^2)\,\epsilon_{yz}\epsilon_{zy} \\ & + \epsilon_{xx}[\beta\gamma\,\epsilon_{zy} + \epsilon_{yz}\,\beta\gamma]\} \end{aligned}$$

$|\hat{\underline{\underline{\epsilon}}}|$ = the value of the determinant of the dielectric tensor

$$D = k_o^2 \Big\{ (\gamma^2+\beta^2)\,[\epsilon_{yy}(\beta^2+\alpha^2) + \epsilon_{zz}(\gamma^2+\alpha^2)] + \epsilon_{xx}(\gamma^2+\alpha^2)(\beta^2+\alpha^2)$$

$$+ \alpha\beta\,[\epsilon_{yz}\alpha\gamma+\epsilon_{zx}\beta\gamma] + \epsilon_{xy}\beta\gamma^2\alpha+\alpha\gamma\,[\alpha\beta\epsilon_{zy}+\beta\gamma\epsilon_{yx}]$$

$$+ \epsilon_{xz}\beta^2\gamma\alpha + \alpha\gamma\,[\epsilon_{zx}(\gamma^2+\alpha^2) - \epsilon_{yy}\alpha\gamma] + \epsilon_{xz}(\gamma^2+\alpha^2)\,\alpha\gamma$$

$$+ \alpha\beta\,[\epsilon_{yx}(\beta^2+\alpha^2) - \epsilon_{zz}\alpha\beta] + \epsilon_{xy}\alpha\beta\,(\beta^2+\alpha^2)$$

$$+ (\gamma^2+\beta^2)\,(\epsilon_{zy}+\epsilon_{yz})\,\beta\gamma-\epsilon_{xx}\beta^2\gamma^2 \Big\}$$

A solution to equation (32) will give

$$K_{1,2} = \left\{ (2D)^{-1} \left[H \pm \sqrt{H^2 - 4\,k_o^6\,D\,|\hat{\underline{\underline{\epsilon}}}|} \right] \right\}^{1/2} \tag{33}$$

where we note two possible choices of sign and, hence, associate the subscripts 1 and 2. Actually there are four solutions, with the possibility of a plus or minus sign in front of each of the K's given by (33). The signs in front of the K's are inconsequential because they only determine the direction of propagation, which will be calculated in a boundary value problem by the actual boundary conditions. The solutions for K_1 and K_2 given by (33) are implicit, since the elements of the dielectric tensor are functions of $\underline{K}$. In order to obtain an explicit solution for K_1 and K_2, we will perturb with respect to ϵ_s. As a first approximation, let ϵ_s equal zero, then K_1 and K_2 are

$$K_1^{(1)} = k \tag{34}$$

$$K_2^{(1)} = k \tag{35}$$

The superscript one indicates a first approximation. A second approximation can be obtained by using (34) and (35) in the dielectric tensor and by letting ϵ_s be finite, but not too large. Then the second approximation can be calculated by using (33). In general, the mean wave in the random medium will have the form

$$\langle \underline{E}\,(\underline{r}) \rangle = \underline{A}_1\, e^{-j\underline{K}_1\cdot\underline{r}} + \underline{A}_2\, e^{-j\underline{K}_2\cdot\underline{r}} \tag{36}$$

$$\underline{K}_1 = K_1\,(\alpha_1\,\underline{a}_x + \beta_1\,\underline{a}_y + \gamma_1\,\underline{a}_z)$$

$$\underline{K}_2 = K_2\,(\alpha_2\,\underline{a}_x + \beta_2\,\underline{a}_y + \gamma_2\,\underline{a}_z)$$

When computing the elements of the dielectric tensor, the direction cosines of each wave will be computed using the first approximation for K_1 and K_2 along with the appropriate boundary conditions.

Each wave in (36) must satisfy equation (31) by itself. This fact can be used to determine the relationships among the electric field components.

$$A_{1x} = a_{1x} A_{1z} \tag{37}$$

$$A_{1y} = a_{1y} A_{1z} \tag{38}$$

$$a_{1x} = \frac{a_{12}\, a_{23} - a_{13}\, a_{22}}{a_{11}\, a_{22} - a_{12}\, a_{21}}$$

$$a_{1y} = \frac{a_{21}\, a_{13} - a_{11}\, a_{23}}{a_{22}\, a_{11} - a_{12}\, a_{21}}$$

The coefficients a_{ij} are the elements of the square matrix in equation (31), with K being set equal to K_1, $\alpha = \alpha_1$, $\beta = \beta_1$, and $\gamma = \gamma_1$. A similar relationship can be written for A_{2x} and A_{2y}.

$$A_{2x} = b_{2x} A_{2z} \tag{39}$$

$$A_{2y} = b_{2y} A_{2z} \tag{40}$$

$$b_{2x} = \frac{b_{12}\, b_{23} - b_{13}\, b_{22}}{b_{11}\, b_{22} - b_{12}\, b_{21}}$$

$$b_{2y} = \frac{b_{21}\, b_{23} - b_{13}\, b_{22}}{b_{11}\, b_{22} - b_{12}\, b_{21}}$$

The coefficients b_{ij} are the elements of the square matrix in equation (31) with K being set equal to K_2. The general equation for the mean wave now becomes

$$\langle \underline{E}(\underline{r}) \rangle = A_{1z} (a_{1x}\, \underline{a}_x + a_{1y}\, \underline{a}_y + \underline{a}_z)\, e^{-j\underline{K}_1 \cdot \underline{r}} +$$

$$A_{2z} (b_{2x}\, \underline{a}_x + b_{2y}\, \underline{a}_y + \underline{a}_z)\, e^{-j\underline{K}_2 \cdot \underline{r}} \tag{41}$$

One could now use the form of the mean wave given by (41) and place it in (20) to determine the scattered wave in a random medium of infinite extent. It should be clear, however, that equation (41) was derived without making any assumptions about the direction of propagation or the form of the correlation function. We also assumed that the three equations given by (31) were coupled. We will now go back to equation (31) and see what happens when the direction of propagation of the mean wave and the correlation function of μ ($\underline{r}$) take on specific forms. If the assumption of dependence is correct, one can see from (41) how depolarization might arise. For instance, suppose a wave that is polarized only in y is incident from free space onto a half space of random media. Then, from (41) the mean wave would generate x and z components as well as a y component, and a depolarized term in the scattered field would result.

Now, let us consider what happens to equation (31) when we assume that the correlation function, B ($\underline{r}$ - $\underline{r}'$), is isotropic. This means that B ($\underline{r}$ - $\underline{r}'$) is a function only of $|\underline{r} - \underline{r}'|$ and is therefore independent of direction. We will also assume that the direction cosine of $\underline{K}$ with respect to the y axis (β) is equal to zero. This assumption is certainly allowable since we can orient our coordinate system in any manner we wish; therefore, we will choose the orientation that sets β equal to zero. By using these assumptions, the element ϵ_{xy} of the dielectric tensor is

$$\epsilon_{xy} = k_o^2 \epsilon_s^2 \int_{v'} B(|\underline{r} - \underline{r}'|) \left\{ \frac{(x - x')(y - y')}{R^2} \right\} f_3 (R) e^{j\underline{K} \cdot (\underline{r} - \underline{r}')} \, \underline{dr}' \tag{42}$$

where $\underline{K} = K (\alpha \, \underline{a}_x + \gamma \, \underline{a}_z)$

Letting x - x' = u, y - y' = v, and letting z - z' = w, and remembering that v' is over all space, equation (42) becomes

$$\epsilon_{xy} = k_o^2 \epsilon_s^2 \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw \, B(R) \left\{ \frac{uv}{R^2} \right\} f_3 (R) e^{j\underline{K} \cdot R} \tag{43}$$

where $R = |\underline{r} - \underline{r}'|$

The upper part of figure 3 shows the relationship between the $\underline{K}$ and $\underline{R}$ vectors in the u, v, w coordinate system. Since $\beta = 0$, the $\underline{K}$ vector is in the u, w plane. We want to rotate the u, v, w coordinate system to u^1, v^1, w^1 in order that the w^1 axis coincides with the direction of $\underline{K}$. The resultant rotated coordinate system is shown in the lower portion of figure 3.

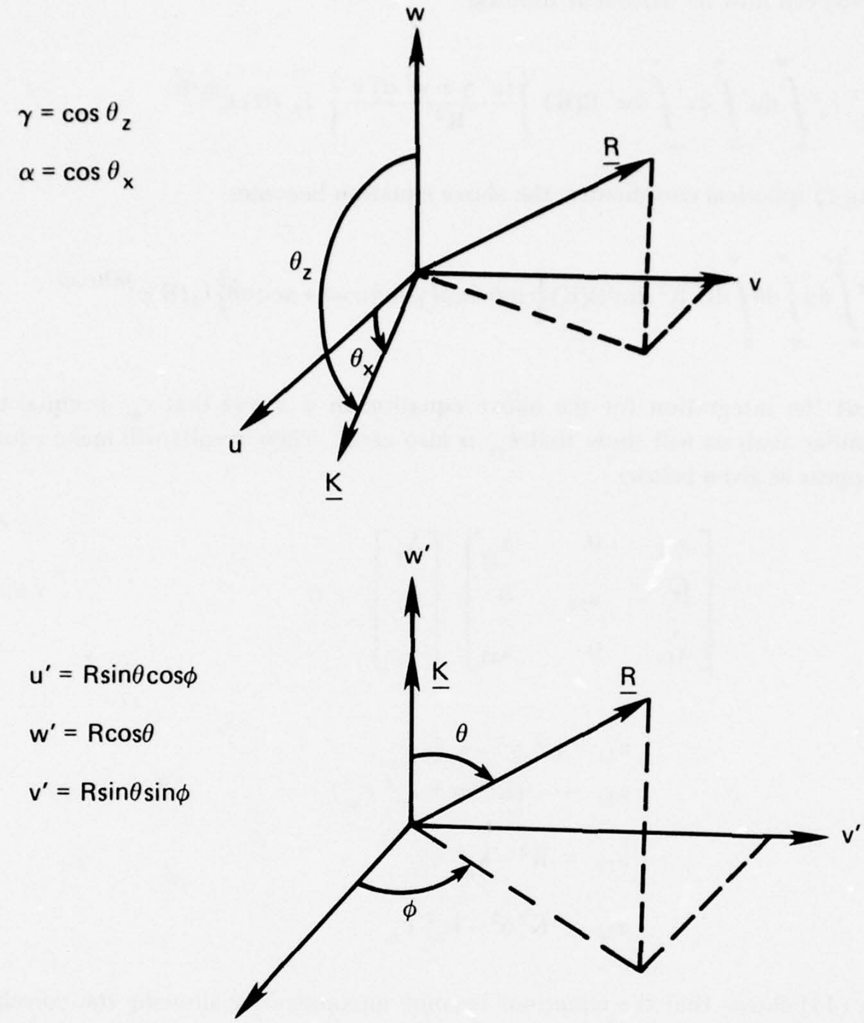

Figure 3. Geometry of $\underline{K}$ and $\underline{R}$ Vectors

The mathematical relationship between the unprimed and primed coordinates are given below:

$$u = u' \cos\theta_z + w' \sin\theta_z = u'\gamma + w'\alpha$$

$$w = -u' \sin\theta_z + w' \cos\theta_z = -u'\alpha + w'\gamma$$

$$v = v'$$

Equation (43) can now be written as follows:

$$\epsilon_{xy} = k_o^2 \epsilon_s^2 \int_{-\infty}^{\infty} du' \int_{-\infty}^{\infty} dv' \int_{-\infty}^{\infty} dw' \, B(R) \left\{ \frac{(u' \gamma + w' \alpha) v'}{R^2} \right\} f_3 (R) e^{j\underline{K}\cdot\underline{R}}$$

By changing to spherical coordinates, the above equation becomes

$$\epsilon_{xy} = k_o^2 \epsilon_s^2 \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{\infty} dR \, R^2 \sin\theta B(R) \left\{ \sin\theta \sin\phi (\gamma \sin\theta \cos\phi + \alpha \cos\theta) \right\} f_3 (R) e^{jKR\cos\theta}$$

Carrying out the integration for the above equation in ϕ shows that ϵ_{xy} is equal to zero. A similar analysis will show that ϵ_{yz} is also zero. These results will make equation (31) appear as given below:

$$\begin{bmatrix} a_{11} & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{13} & 0 & a_{33} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = 0 \qquad (44)$$

where

$$a_{11} = K^2 \gamma^2 - k_o^2 \epsilon_{xx}$$
$$a_{13} = - (K^2 \gamma\alpha + k_o^2 \epsilon_{xz})$$
$$a_{22} = K^2 - k_o^2 \epsilon_{yy}$$
$$a_{33} = K^2 \alpha^2 - k_o^2 \epsilon_{zz}$$

Expression (44) shows that the equations become uncoupled by allowing the correlation function to be isotropic. This means that the equation derived for the mean wave earlier (41) is not valid for an isotropic correlation function and that a new mean wave must be sought. For a wave polarized in y such that $A_y \neq 0$, we must have $a_{22} = 0$. This condition yields the following solution for K, which we designate K_{yy}.

$$K_{yy}^2 = k_o^2 \epsilon_{yy}$$

$$K_{yy} = k \left\{ 1 + k_o^4 \epsilon_s^2 L/k^2 \right\}^{1/2} \qquad (45)$$

$$L = \int_{v'} B(R) \left\{ f_2 (R) + \frac{(y - y')^2}{R^2} f_3 (R) \right\} e^{j\underline{K}\cdot(\underline{r} - \underline{r}')} \, d\underline{r}'$$

The definition of L given above comes from ϵ_{yy}. The value of $\underline{K}$ to be used in L when computing (45) would be $\underline{K}_{yy}$. The volume integral is evaluated in appendix A along with the other elements of the dielectric tensor and the direction cosines α and γ. It can also be seen from equation (45) that a first approximation for K_{yy} (which occurs when ϵ_s is equal to zero) will simply be k. We will rewrite K_{yy} as follows:

$$K_{yy} = k\,(x_1 + jy_1)^{1/2}$$

$$x_1 = \mathrm{Re}\left\{1 + k_o^4\,\epsilon_s^2\,L/k^2\right\}$$

$$y_1 = \mathrm{Im}\left\{1 + k_o^4\,\epsilon_s^2\,L/k^2\right\}$$

By changing to polar coordinates, we have

$$K_{yy} = k\,\sqrt{\rho_1}\,\exp\,(j\phi_1/2) \tag{46}$$

$$\rho_1 = \sqrt{x_1^{\,2} + y_1^{\,2}}$$

$$\phi_1 = \tan^{-1}\,(y_1/x_1) \qquad -\pi < \phi_1 < \pi$$

Equation (46) represents an expression for the effective propagation constant when the mean wave is polarized in the y direction.

If we allow the mean wave to be polarized in the xz plane such that A_x and A_z are nonzero, then the following determinant must be set equal to zero in order to obtain a nontrivial solution for K:

$$\det \begin{vmatrix} a_{11} & a_{13} \\ a_{13} & a_{33} \end{vmatrix} = a_{11}\,a_{33} - a_{13}^{\;2} = 0$$

The above equation will yield the following solution for K, which we designate K_{xz}:

$$K_{xz} = k_o \left\{ \frac{\epsilon_{xx}\,\epsilon_{zz} - \epsilon_{xz}^{\;2}}{\alpha^2\,\epsilon_{xx} + \gamma^2\,\epsilon_{zz} + 2\gamma\alpha\epsilon_{xz}} \right\}^{1/2} \tag{47}$$

As a first approximation for K_{xz}, we set ϵ_s equal to zero. When this is done, the following result is obtained:

$$K_{xz}^{(1)} = k$$

The first approximation for K_{xz} is the same as the first approximation for K_{yy}. When the first approximation for K_{xz} is used to compute the elements of the dielectric tensor, a second approximation for K_{xz} can be computed from equation (47), which we now write in a different form.

$$K_{xz} = k_o \, (x_2 + jy_2)^{1/2}$$

$$x_2 = \mathrm{Re}\left\{\frac{\epsilon_{xx}\,\epsilon_{zz} - \epsilon_{xz}^{\;2}}{\alpha^2\,\epsilon_{xx} + \gamma^2\,\epsilon_{zz} + 2\gamma\,\alpha\,\epsilon_{xz}}\right\}$$

$$y_2 = \mathrm{Im}\left\{\frac{\epsilon_{xx}\,\epsilon_{zz} - \epsilon_{xz}^{\;2}}{\alpha^2\,\epsilon_{xx} + \gamma^2\,\epsilon_{zz} + 2\gamma\alpha\epsilon_{xz}}\right\}$$

By changing to polar coordinates, we have:

$$K_{xz} = k_o \sqrt{\rho_2} \;\; e^{j\phi_2/2} \tag{48}$$

$$\rho_2 = \sqrt{x_2^{\;2} + y_2^{\;2}}$$

$$\phi_2 = \tan^{-1}\,(y_2/x_2) \qquad -\pi < \phi_2 < \pi$$

We have obtained two solutions for the effective propagation constant. One solution is used when the mean wave is polarized in the y direction. The other solution is used when the mean wave is polarized in the xz plane. The next section of this report will consider the problem of a horizontally polarized wave incident obliquely at an angle θ_i from a free space medium ($z > 0$) unto a lossy random medium ($z < 0$). The boundary conditions for the mean wave will be satisfied; then the scattered field in the free space medium will be calculated. Finally, an equation for the radar backscatter coefficient will be derived. Following this, a similar derivation will be performed for the case where the incident wave is vertically polarized.

Horizontal Polarization Analysis. The results of the previous section on Scattering Geometry and Wave Propagation In a Random Medium of Infinite Extent, will now be used to derive a radar backscatter coefficient for the case of an incident wave which is horizontally polarized. Consider figure 4, which shows a plane wave incident from free space ($z > 0$) at angle θ_i onto a half space ($z < 0$) of random media. The polarization of the incident wave is in the +y direction. The coordinate system is right-handed so that the +y direction is into the plane of the paper.

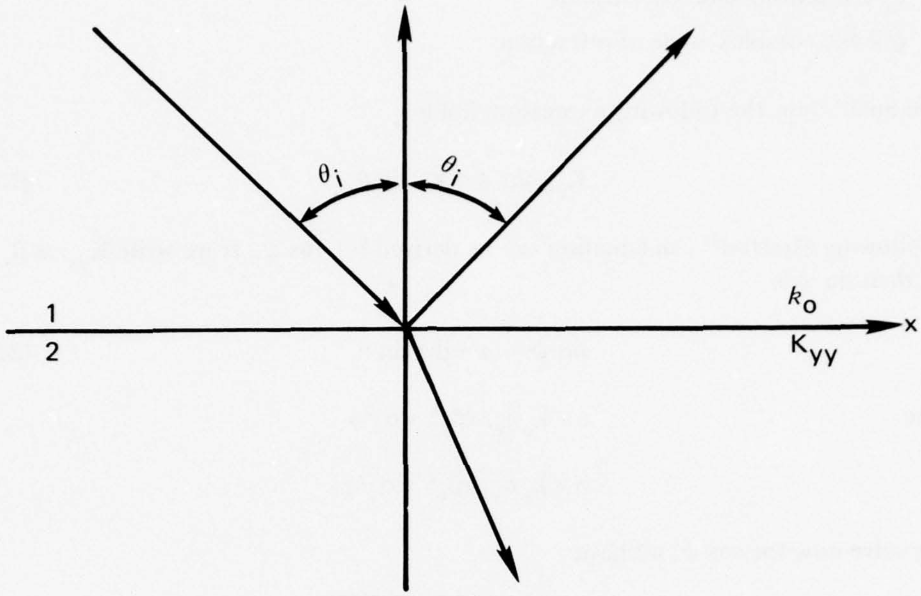

Figure 4. Geometry for the Coherent Waves (Horizontal Polarization)

The solutions for the mean waves in the two regions will be considered first. The scattered waves will be considered later. The total mean electric field ($\underline{E}_1$) in the upper medium can be written as

$$\underline{E}_1 = \underline{a}_y \left[e^{-jk_o \underline{n}_i \cdot \underline{r}} + R_\perp e^{-jk_o \underline{n}_r \cdot \underline{r}} \right] \tag{49}$$

$$\underline{n}_i = \underline{a}_x \sin\theta_i - \underline{a}_z \cos\theta_i$$

$$\underline{n}_r = \underline{a}_x \sin\theta_i + \underline{a}_z \cos\theta_i$$

$$\underline{r} = x\,\underline{a}_x + y\,\underline{a}_y + z\,\underline{a}_z$$

$R_\perp$ is a reflection coefficient.

The first term in (49) represents the incident wave, and the second term represents the reflected wave. The mean electric field ($\underline{E}_2$) in the lower medium can be written as

$$\underline{E}_2 = \underline{a}_y\, T_\perp\, e^{-j K_{yy} \underline{n}_3 \cdot \underline{r}} \tag{50}$$

$$\underline{n}_3 = \underline{a}_x \sin\psi - \underline{a}_z \cos\psi$$

$T_\perp$ is a transmission coefficient.

ψ is the complex angle of refraction.

From Snell's law, the following expression holds:

$$K_{yy} \sin \psi = k_o \sin \theta_i \tag{51}$$

By following Stratton[21], an equation can be derived for cos ψ. If we write K_{yy} as β_y - $j\alpha_y$, then sin ψ is

$$\sin \psi = (a + jb) \sin \theta_i \tag{52}$$

where $$a = k_o\beta_y / (\beta_y^2 + \alpha_y^2)$$

and $$b = k_o\alpha_y / (\beta_y^2 + \alpha_y^2)$$

If we solve now for cos ψ, we have

$$\cos \psi = \sqrt{1 - (a + jb)^2 \sin^2 \theta_i} = \rho_t e^{-j\phi_t} \tag{53}$$

The magnitude ρ_t and the phase ϕ_t can be found by squaring (53) and equating real and imaginary parts on either side of the equation. The subscript t has been used to indicate reference to the mean wave, which is transmitted into the random media.

$$\rho_t^2 \cos 2\phi_t = 1 + (b^2 - a^2) \sin^2 \phi_i \tag{54}$$

$$\rho_t^2 \sin 2\phi_t = 2ab \sin^2 \phi_i \tag{55}$$

Solving equations (54) and (55) for ρ_t and ϕ_t, we have

$$\rho_t = \left\{ [1 + (b^2 - a^2) \sin^2 \theta_i]^2 + 4a^2 b^2 \sin^4 \theta_i \right\}^{1/4} \tag{56}$$

$$\phi_t = \tfrac{1}{2} \tan^{-1} \left\{ \frac{2ab \sin^2 \theta_i}{1 + (b^2 - a^2) \sin^2 \theta_i} \right\} \tag{57}$$

An expression can now be written for K_{yy} cos ψ as follows:

$$K_{yy} \cos \psi = (\beta_y - j\alpha_y)\, \rho_t\, (\cos \phi_t - j\sin\phi_t)$$
$$K_{yy} \cos \psi = q - jp \tag{58}$$

[21] Julius Adams Stratton, *Electromagnetic Theory*, McGraw Hill, 1941, page 501.

where $$p = \rho_t \ (\beta_y \sin \phi_t + \alpha_y \cos \phi_t) \tag{59}$$

and $$q = \rho_t \ (\beta_y \cos \phi_t - \alpha_y \sin \phi_t) \tag{60}$$

The mean electric field in the random medium can now be written as

$$\underline{E}_2 \ (\underline{r}) = \underline{a}_y \ T_\perp \ e^{-jk_o x \sin\theta_i} \ e^{jqz} \ e^{pz} \tag{61}$$

The tangential components of the electric and magnetic fields must be continuous across the boundary at z = 0. The magnetic fields in each medium can be computed by using the electric fields and the Maxwell equation $\nabla \times \underline{E} = - j\omega\mu_o \underline{H}$. The boundary conditions will permit a solution for $R_\perp$ and $T_\perp$.

$$R_\perp = \frac{k_o \cos \theta_i - K_{yy} \cos \psi}{k_o \cos \theta_i + K_{yy} \cos \psi} \tag{62}$$

$$T_\perp = \frac{2 \, k_o \cos \theta_i}{K_{yy} \cos \psi + k_o \cos \theta_i} \tag{63}$$

We are now in a position to compute the scattered fields and then to calculate the radar backscatter coefficient using the scattered field in the upper medium (free space). The equation for the scattered electric field in the random medium is

$$\mathcal{L} \, \underline{E}_s = \xi < \underline{E} >$$

$$[\nabla(\nabla \cdot \underline{E}_s) - \nabla^2 \underline{E}_s - k^2 \underline{E}_s] = k_o^2 \, \epsilon_s \mu \, (\underline{r}) \, \underline{a}_y T_\perp \, e^{-jk_o x \sin\theta_i} e^{jqz} e^{pz} \tag{64}$$

If we say that $\underline{E}_s$ is of order ϵ_s in magnitude, then $\nabla \cdot \underline{E}_s$ can be shown to be of order ϵ_s^2 and can be neglected in (64), since every other term in the equation is of order ϵ_s. The three components of (64) are

$$\nabla^2 E_{sx} + k^2 E_{sx} = 0 \tag{65}$$

$$\nabla^2 E_{sy} + k^2 E_{sy} = k_o^2 \, \epsilon_s \, \mu \, (r) \, T_\perp \, e^{-jk_o x \sin\theta_i} e^{jqz} e^{pz} \tag{66}$$

$$\nabla^2 E_{sz} + k^2 E_{sz} = 0 \tag{67}$$

Solutions for E_{sx} and E_{sz} can be written in the form of a Fourier transform as follows:

$$E_{sx}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_x(k_x, k_y)\, e^{jk_z'z}\, e^{jk_x x}\, e^{jk_y y}\, dk_x dk_y \tag{68}$$

and

$$E_{sz}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_z(k_x, k_y)\, e^{jk_z'z}\, e^{jk_x x}\, e^{jk_y y}\, dk_x dk_y \tag{69}$$

In the above equations, B_x and B_z are functions of the Fourier variables k_x and k_y. A solution for k_z' can easily be obtained by putting (69) back into (67).

$$k_z' = \sqrt{k^2 - k_x^2 - k_y^2} \tag{70}$$

Since k is complex, equation (70) is not in a good form for calculations. Letting $k = \beta_o - j\alpha_o$, we have

$$k_z'^2 = \beta_o^2 - 2j\beta_o\alpha_o - \alpha_o^2 - k_x^2 - k_y^2 = \rho_z e^{j\phi_z} \tag{71}$$

The magnitude ρ_z and phase ϕ_z of $k_z'^2$ can be found by squaring (71) and equating real and imaginary parts on either side of the equation.

$$\rho_z = \sqrt{(\beta_o^2 - \alpha_o^2 - k_x^2 - k_y^2)^2 + 4\alpha_o^2\beta_o^2} \tag{72}$$

$$\phi_z = \tan^{-1}\left[\frac{-2\alpha_o\beta_o}{(\beta_o^2 - \alpha_o^2 - k_x^2 - k_y^2)}\right] \tag{73}$$

If k_z' is now written as $k_r + jk_i$ where k_r and k_i are both real, we have

$$k_z' = k_r + j\,k_i \tag{74}$$

$$k_r = \rho_z^{1/2}\cos(\phi_z/2)$$

$$k_i = \rho_z^{1/2}\sin(\phi_z/2)$$

A solution for E_{sy} is much more difficult than for E_{sx} and E_{sz} owing to the term appearing on the right hand side of equation (66). This term is a random function of all three coordinates owing to μ ($\underline{r}$). A general form for the solution of E_{sy} can be written as

$$E_{sy}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_y(k_x, k_y, z) \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{75}$$

Placing equation (75) into (66) will give

$$\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ -k_x^2 B_y - k_y^2 B_y + \frac{d^2 B_y}{dz^2} + k^2 B_y \right\} \exp(jk_x x + jk_y y)\, dk_x\, dk_y =$$

$$-k_o^2\, \epsilon_s\, \mu(\underline{r})\, T_\perp\, e^{-jk_o x \sin\theta_i}\, e^{jqz}\, e^{pz} \tag{76}$$

We will now define the term $S(k_x, k_y, z)$ as follows:

$$S(k_x, k_y, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(\underline{r})\, e^{-jk_o x \sin\theta_i} \exp(-jk_x x - jk_y y)\, dy\, dx \tag{77}$$

$$\mu(\underline{r})\, e^{-jk_o x \sin\theta_i} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(k_x, k_y, z) \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{78}$$

The assumption has been made that $\mu(\underline{r})$ is Fourier transformable. Using (78) in (76) will give

$$\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ -k_x^2 B_y - k_y^2 B_y + \frac{d^2 B_y}{dz^2} + k^2 B_y + k_o^2\, \epsilon_s\, S(k_x, k_y, z)\, T_\perp\, e^{jqz}\, e^{pz} \right\} \times$$

$$\exp(jk_x x + jk_y y)\, dk_x\, dk_y = 0 \tag{79}$$

One way to be certain that the integral on the left-hand side of (79) is always zero is to set the integrand equal to zero. When this is done, the following second order differential equation will result:

$$\frac{d^2 B_y}{dz^2} + (k^2 - k_x^2 - k_y^2)\, B_y = -k_o^2\, \epsilon_s\, T_\perp\, S(k_x, k_y, z)\, e^{jqz}\, e^{pz} \tag{80}$$

We will let B_y be the sum of a complementary solution (B_{yc}) and of a particular solution (B_{yp}). The complementary solution can be found by setting the right side of (80) equal to zero.

$$B_{yc} = B_{y1}\, e^{jk_z' z} + B_{y2}\, e^{-jk_z' z} \tag{81}$$

The quantities B_{y1} and B_{y2} are independent of z. The form of the particular solution can be written as

$$B_{yp} = v_1(z)\, e^{jk_z'z} + v_2(z)\, e^{-jk_z'z} \tag{82}$$

Placing (82) into (80) and using the method of variation of parameters will yield

$$\frac{dv_1}{dz} = -\frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'}\, S(k_x, k_y, z)\, e^{jqz}\, e^{pz}\, e^{-jk_z'z} \tag{83}$$

$$\frac{dv_2}{dz} = \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'}\, S(k_x, k_y, z)\, e^{jqz}\, e^{pz}\, e^{jk_z'z} \tag{84}$$

Integrating both sides of (83) between the limits of a and z will give

$$v_1(z) = \frac{-k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_a^z S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{-jk_z'\xi}\, d\xi + v_1(a) \tag{85}$$

where the lower limit a is a constant. Integrating equation (84) between the limits of b and z will give

$$v_2(z) = \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_b^z S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{jk_z'\xi}\, d\xi + v_2(b) \tag{86}$$

where b is a constant.

The solution for E_{sy} can now be written as follows:

$$E_{sy}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Bigg\{ B_{y1}\, e^{jk_z'z} + B_{y2}\, e^{-jk_z'z} + \Bigg[v_1(a) - \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_a^{-\infty} S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{-jk_z'\xi}\, d\xi - \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_{-\infty}^{z} S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{-jk_z'\xi}\, d\xi \Bigg] \times$$

$$e^{jk_z'z} + \Bigg[v_2(b) + \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_b^{-\infty} S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{jk_z'\xi}\, d\xi + \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_{-\infty}^{z} S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi} \times$$

$$e^{jk_z'\xi}\, d\xi \Bigg] e^{-jk_z'z} \Bigg\} \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{87}$$

where we have made the integral in v_1 (z), a sum of two integrals. Similarly, v_2 (z) has been made equal to the sum of two integrals. We will now define the coefficients B_1 and B_2 as

$$B_1 = B_{y1} + v_1(a) - \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_a^{-\infty} S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{-jk_z'\xi}\, d\xi$$

$$B_2 = B_{y2} + v_1(b) + \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \int_b^{-\infty} S(k_x, k_y, \xi)\, e^{jq\xi}\, e^{p\xi}\, e^{jk_z'\xi}\, d\xi$$

Incorporating the above definitions into equation (87) will yield

$$E_{sy}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Bigg\{ B_1\, e^{jk_z'z} + B_2\, e^{-jk_z'z} - \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \left[\int_{-\infty}^{z} S(k_x, k_y, \xi)\, f(\xi)\, e^{-jk_z'\xi}\, d\xi \right] e^{jk_z'z}$$

$$+ \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \left[\int_{-\infty}^{z} S(k_x, k_y, \xi)\, f(\xi)\, e^{jk_z'\xi}\, d\xi \right] e^{-jk_z'z} \Bigg\} \exp(jk_x x + jk_y y)\, dk_x\, dk_y \qquad (88)$$

where $f(\xi) = e^{jq\xi}\, e^{p\xi}$.

The first term in the integrand of equation (88) represents a set of waves propagating in the minus z direction. The second term represents waves propagating in the plus z direction. From the radiation condition, E_{sy} must go to zero as z approaches minus infinity. This means that B_2 must be zero. The rest of the terms in the integrand do go to zero as z approaches minus infinity, although the last term does so only by using L'Hospital's rule. The final expression for E_{sy} is then

$$E_{sy}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Bigg\{ B_1\, e^{jk_z'z} - \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \left[\int_{-\infty}^{z} S(k_x, k_y, \xi)\, f(\xi)\, e^{-jk_z'\xi}\, d\xi \right] e^{jk_z'z}$$

$$+ \frac{k_o^2 \epsilon_s T_\perp}{2j\, k_z'} \left[\int_{-\infty}^{z} S(k_x, k_y, \xi)\, f(\xi)\, e^{jk_z'\xi}\, d\xi \right] e^{-jk_z'z} \Bigg\} \exp(jk_x x + jk_y y)\, dk_x\, dk_y \qquad (89)$$

The scattered electric field ($\underline{E}_s'$) in the upper medium ($z > 0$) must satisfy the following homogeneous wave equation:

$$\nabla^2 \underline{E}_s' + k_o^2 \underline{E}_s' = 0$$

The components of $\underline{E}_s'$ can be written as

$$E_{sx}'(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_x(k_x, k_y) \exp(jk_x x + jk_y y - jk_z z) \, dk_x dk_y \tag{90}$$

$$E_{sy}'(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_y(k_x, k_y) \exp(jk_x x + jk_y y - jk_z z) \, dk_x dk_y \tag{91}$$

$$E_{sz}'(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_z(k_x, k_y) \exp(jk_x x + jk_y y - jk_z z) \, dk_x dk_y \tag{92}$$

There are six unknowns associated with the components of the scattered electric fields in the two mediums. These unknowns are A_x, A_y, A_z, B_x, B_1, and B_z. Therefore, six independent equations must be developed that will allow for the solution of the six unknowns. Actually, we are only interested in solutions for A_x, A_y, and A_z, which will be used to compute the far zone scattered field. At $z = 0$, the tangential components of the electric and magnetic scattered fields must be continuous. This can be stated as

$$\underline{a}_z \times [\underline{E}_s - \underline{E}_s'] = 0 \tag{93}$$

$$\underline{a}_z \times [\underline{H}_s - \underline{H}_s'] = 0 \tag{94}$$

where $\underline{H}_s'$, and $\underline{H}_s$ are the scattered magnetic fields in the upper medium and in the random medium, respectively. The two boundary conditions given above can be restated in terms of the electric field components as

$$E_{sx}'(x,y,0) = E_{sx}(x,y,0) \tag{95}$$

$$E_{sy}'(x,y,0) = E_{sy}(x,y,0) \tag{96}$$

$$\frac{\partial E_{sz}'}{\partial y} - \frac{\partial E_{sy}'}{\partial z} = \frac{\partial E_{sz}}{\partial y} - \frac{\partial E_{sy}}{\partial z} \quad \text{at } z = 0 \tag{97}$$

$$\frac{\partial E_{sx}'}{\partial z} - \frac{\partial E_{sz}'}{\partial x} = \frac{\partial E_{sx}}{\partial z} - \frac{\partial E_{sz}}{\partial x} \quad \text{at } z = 0 \tag{98}$$

In addition to the four equations given above, there are two divergence conditions that can be used. The divergence of the scattered electric field in the upper medium is zero.

$$\nabla \cdot \underline{E}_s' = 0$$

$$k_x A_x + k_y A_y - k_z A_z = 0 \tag{99}$$

The divergence of the scattered electric field in the random medium will be zero, provided we use only the complementary solution for E_{sy}. If we consider only the complementary solution for E_{sy}, then the scattered field in the random medium obeys a homogeneous wave equation of the form

$$\nabla^2 \underline{E}_{sc} + k^2 \underline{E}_{sc} = 0$$

The subscript c has been added to indicate that only complementary solutions are to be considered. The above wave equation implies that

$$\nabla \cdot \underline{E}_{sc} = 0$$

$$k_x B_x + k_y B_1 + k_z' B_z = 0 \tag{100}$$

Evaluating equations (68) and (90) at z = 0 and placing the results in the boundary condition given by (95) will yield

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \{A_x - B_x\} \exp(jk_x x + jk_y y)\, dk_x\, dk_y = 0$$

Taking the Fourier transform of the above equation will give

$$A_x = B_x \tag{101}$$

When equation (89) for E_{sy} (x,y,z) and equation (91) for E_{sy}' (x,y,z) are evaluated at z = 0 and placed into the boundary condition given by (96), the following equation results:

$$\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Bigg\{ A_y - B_1 + \frac{k_o^2\, \epsilon_s\, T_\perp}{2j\, k_z'} \left[\int_{-\infty}^{0} S(k_x, k_y, \xi)\, f(\xi)\, e^{-jk_z' \xi}\, d\xi \right]$$

$$- \frac{k_o^2\, \epsilon_s\, T_\perp}{2j\, k_z'} \left[\int_{-\infty}^{0} S(k_x, k_y, \xi)\, f(\xi)\, e^{jk_z' \xi}\, d\xi \right] \Bigg\} \exp(jk_x x + jk_y y)\, dk_x dk_y = 0$$

Taking the Fourier transform of the above expression will give

$$A_y - B_1 + \frac{k_o^2 \epsilon_s T_\perp}{2j k_z'} (I_1 - I_2) = 0 \qquad (102)$$

where $I_1 = \int_{-\infty}^{0} S(k_x, k_y, \xi) f(\xi) e^{-jk_z' \xi} d\xi$

and $I_2 = \int_{-\infty}^{0} S(k_x, k_y, \xi) f(\xi) e^{jk_z' \xi} d\xi$

Substituting the proper equations for the electric fields into the boundary condition given by (97) and evaluating the final result at z = 0 will give the following expression:

$$\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{ jk_y A_z + jk_z A_y - jk_y B_z + jk_z' B_1 - \frac{k_o^2 \epsilon_s T_\perp}{2} I_1 \right.$$

$$\left. - \frac{k_o^2 \epsilon_s T_\perp}{2} I_2 \right\} \exp(jk_x x + jk_y y)\, dk_x dk_y = 0$$

When we take the Fourier transform of the equation above, the following result is obtained:

$$k_y A_z + k_z A_y - k_y B_z + k_z' B_1 = k_o^2 \epsilon_s T_\perp (I_1 + I_2)/2j \qquad (103)$$

Substituting the proper equations for the electric fields into the boundary condition given by (98) and evaluating the result at z = 0 will give the following equation:

$$\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{- jk_z A_x - jk_x A_z - jk_z' B_x + jk_x B_z\} \exp(jk_x x + jk_y y)\, dk_x\, dk_y = 0$$

Taking the Fourier transform of the above equation will give

$$k_z A_x + k_x A_z + k_z' B_x - k_x B_z = 0 \qquad (104)$$

Equations (99) through (104) are six equations with six unknowns. The quantities of interest are A_x, A_y, and A_z. When these quantities are solved for, the following expressions result:

$$A_x(k_x, k_y) = A_{x1}(k_x, k_y) I_1 + A_{x2}(k_x, k_y) I_2 \quad (105)$$

where $$A_{x1}(k_x, k_y) = -\frac{k_o^2 \epsilon_s T_\perp}{2j}\left[\frac{k_x k_y k_z}{k_z'(k_x^2 + k_y^2 + k_z k_z')(k_z + k_z')}\right]$$

and $$A_{x2}(k_x, k_y) = \frac{k_o^2 \epsilon_s T_\perp}{2j}\left[\frac{k_x k_y (k_z^2 - 2k_z'^2 - k_z k_z')}{k_z'(k_x^2 + k_y^2 + k_z k_z')(k_z + k_z')^2}\right]$$

$$A_y(k_x, k_y) = A_{y1}(k_x, k_y) I_1 + A_{y2}(k_x, k_y) I_2 \quad (106)$$

where $$A_{y1}(k_x, k_y) = -\frac{k_o^2 \epsilon_s T_\perp}{2j}\left[\frac{k_y^2 k_z}{k_z'(k_x^2 + k_y^2 + k_z k_z')(k_z + k_z')}\right]$$

and $$A_{y2}(k_x, k_y) = \frac{k_o^2 \epsilon_s T_\perp}{2j}\left[\frac{k_y^2 k_z + 2k_z'(k_x^2 + k_z k_z')}{k_z'(k_x^2 + k_y^2 + k_z k_z')(k_z + k_z')}\right]$$

$$A_z(k_x, k_y) = A_{z1}(k_x, k_y) I_1 + A_{z2}(k_x, k_y) I_2 \quad (107)$$

where $$A_{z1}(k_x, k_y) = [k_x A_{x1}(k_x, k_y) + k_y A_{y1}(k_x, k_y)]/k_z$$

and $$A_{z2}(k_x, k_y) = [k_x A_{x2}(k_x, k_y) + k_y A_{y2}(k_x, k_y)]/k_z$$

A solution for the scattered electric field ($\underline{E}_s'$) in the upper medium has now been obtained in terms of the integrals I_1 and I_2, and also in terms of the integrals in k_x and k_y. In order to calculate the scattered far field for the case of backscatter, we will use the Stratton-Chu integral as modified by Silver.[22] The Stratton-Chu integral will be changed to a form that will allow the results of the previous derivation to be utilized in computing the far field. The scattered far field ($\underline{E}_{sf}$) in the direction defined by the unit vector $\underline{n}_2$ can be stated as

$$\underline{E}_{sf} = \hat{K}\, \underline{n}_2 \times \int_s [\underline{n} \times \underline{E}_s' - \eta\, \underline{n}_2 \times (\underline{n} \times \underline{H}_s')]\, e^{jk_o \underline{n}_2 \cdot \underline{r}}\, ds \quad (108)$$

where $\underline{r}$ is a position vector pointing from the origin of the (x,y,z) coordinate system to a surface element ds on the surface $z = 0$; $\hat{K} = -jk_o \exp(-jk_o R_o)/4\pi R_o$; the distance

22 Samuel Silver, *Microwave Antenna Theory and Design*, New York, McGraw Hill, 1947, p. 161.

R_o is from the origin to the field point where $\underline{E}_{sf}$ is calculated; $\underline{E}_s'$ and $\underline{H}_s'$ are the total scattered electric and magnetic fields evaluated on the surface $z = 0$; η is the intrinsic impedance of free space (upper medium); and $\underline{n}$ is a unit vector normal to the surface $z = 0$. The surface integral in (108) is over the illuminated area. For the case of backscatter, $\underline{n}_2$ is

$$\underline{n}_2 = -\underline{a}_x \sin\theta_i + \underline{a}_z \cos\theta_i = -\underline{n}_i$$

The magnetic field components of $\underline{H}_s'$ can be computed using equations (90 to 92) along with the Maxwell equation $\nabla \times \underline{E}_s' = -j\omega\mu_o \underline{H}_s'$. When this is done, the following components of $\underline{H}_s'$ are obtained:

$$\eta H_{sx}' = \frac{-1}{(2\pi)^2 k_o} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_y A_z + k_z A_y) \exp(jk_x x + jk_y y - jk_z z)\, dk_x dk_y \tag{109}$$

$$\eta H_{sy}' = \frac{1}{(2\pi)^2 k_o} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_z A_x + k_x A_z) \exp(jk_x x + jk_y y - jk_z z)\, dk_x dk_y \tag{110}$$

$$\eta H_{sz}' = \frac{1}{(2\pi)^2 k_o} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (k_y A_x - k_x A_y) \exp(jk_x x + jk_y y - jk_z z)\, dk_x dk_y \tag{111}$$

The unit normal ($\underline{n}$) to the surface $z = 0$ is simply the unit vector in the z direction, which is $\underline{a}_z$. The calculation of the quantities $\underline{n}_2 \times (\underline{n} \times \underline{E}_s')$ and $\underline{n}_2 \times (\underline{n}_2 \times (\underline{n} \times \underline{H}_s'))$ are now straight forward.

$$\underline{n}_2 \times (\underline{n} \times \underline{E}_s') = -(\underline{a}_x E_{sx}' \cos\theta_i + \underline{a}_y E_{sy}' \cos\theta_i + \underline{a}_z E_{sx}' \sin\theta_i) \tag{112}$$

$$\underline{n}_2 \times (\underline{n}_2 \times (\underline{n} \times \underline{H}_s')) = \underline{a}_x H_{sy}' \cos^2\theta_i - \underline{a}_y H_{sx}' + \underline{a}_z H_{sy}' \sin\theta_i \cos\theta_i \tag{113}$$

Placing equations (90 to 92) and equations (109 to 111) into (112) and (113) and evaluating the result at $z = 0$ will give the following result for the scattered far field ($\underline{E}_{sf}$):

$$\underline{E}_{sf} = \frac{-\hat{K}}{(2\pi)^2} \iint_S \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Big\{ \underline{a}_x \left[A_x \cos\theta_i + \cos^2\theta_i (k_z A_x + k_x A_z)/k_o\right] +$$

$$+ \underline{a}_y \left[A_y \cos\theta_i + (k_y A_z + k_z A_y)/k_o\right] + \underline{a}_z \left[A_x \sin\theta_i + \sin\theta_i \cos\theta_i (k_z A_x +\right.$$

$$\left. k_x A_z)/k_o\right] \Big\} \exp(jk_x x + jk_y y - jk_o x \sin\theta_i)\, dy\, dx\, dk_x dk_y \tag{114}$$

We will now allow the limits on the integrals in x and y to be from minus infinity to plus infinity. This will be a reasonable assumption as long as the dimensions of the actual illuminated area are all much greater than the correlation distance of the random dielectric fluctuations. The integrals in x and y will then yield Dirac delta functions.

$$\underline{E}_{sf} = -\hat{K}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big\{\underline{a}_x[A_x\cos\theta_i + \cos^2\theta_i\,(k_zA_x + k_xA_z)/k_o] + \underline{a}_y[A_y\cos\theta_i +$$

$$(k_yA_z + k_zA_y)/k_o] + \underline{a}_z\,[A_x\sin\theta_i + \sin\theta_i\cos\theta_i\,(k_zA_x + k_xA_z)/k_o]\Big\}\,\cdot$$

$$\delta(k_x - k_o\sin\theta_i)\,\delta(k_y)\,dk_x\,dk_y \tag{115}$$

Evaluating the k_x and k_y integrals in (115) and simplifying the algebra will give the following equation for $\underline{E}_{sf}$:

$$\underline{E}_{sf} = \frac{2\cos\theta_i(jk_o)\,e^{-jk_oR_o}}{4\pi R_o}\,[\underline{a}_xA_x(k_o\sin\theta_i,0) + \underline{a}_yA_y(k_o\sin\theta_i,0) + \underline{a}_zA_z(k_o\sin\theta_i,0)] \tag{116}$$

If the incident wave is horizontally polarized and the received wave is also horizontally polarized, then our interest is only in the y component of $\underline{E}_{sf}$. We designate the y component of $\underline{E}_{sf}$ as E_{HH}, where the letter subscripts refer to the polarization of the incident and the received waves respectively.

$$E_{HH} = \frac{2\cos\theta_i(jk_o)e^{-jk_oR_o}}{4\pi R_o}\,A_y(k_o\sin\theta_i,0) \tag{117}$$

The definition of the radar backscatter coefficient can be stated as

$$\sigma^o_{HH} = \lim_{R_o\to\infty}\frac{4\pi R_o^2}{A_o}\,\frac{<E_{HH}E^*_{HH}>}{\underline{E}_i\cdot\underline{E}^*_i} \tag{118}$$

where $\underline{E}_i$ is the incident wave and a * is used to denote a complex conjugate. The brackets around $E_{HH}\ E^*_{HH}$ are used to indicate the computation of a statistical average. The quantity A_o is the illuminated area. By using the computed form of A_y as given by (106), E_{HH} can be written as

$$E_{HH} = \frac{2jk_o e^{-jk_oR_o} \cos\theta_i}{4\pi R_o} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{0} d\xi\, \mu(\underline{r}) e^{-2jk_o x \sin\theta_i} e^{jq\xi} e^{p\xi}$$

$$\cdot \left\{ A_{y1}(k_o \sin\theta_i, 0) e^{-jk_z'\xi} + A_{y2}(k_o \sin\theta_i, 0) e^{jk_z'\xi} \right\} \qquad (119)$$

where the definitions of I_1, I_2 and $S(k_o \sin\theta_i, 0, \xi)$ have been used in (119). The complex conjugate of (119) can now be written easily as

$$E_{HH}^* = \frac{-2jk_o e^{jk_oR_o} \cos\theta_i}{4\pi R_o} \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{0} d\xi'\, \mu(\underline{r}') e^{2jk_o x' \sin\theta_i} e^{-jq\xi'} e^{p\xi'}$$

$$\cdot \left\{ A_{y1}^*(k_o \sin\theta_i, 0)\, e^{jk_z'^*\xi'} + A_{y2}^*(k_o \sin\theta_i, 0)\, e^{-jk_z'^*\xi'} \right\} \qquad (120)$$

An expression for the average of $E_{HH}\, E_{HH}^*$ can now be written.

$$< E_{HH} E_{HH}^* > = \frac{k_o^2 \cos^2\theta_i}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \int_{-\infty}^{0} d\xi \int_{-\infty}^{0} d\xi' <\mu(\underline{r})\mu(\underline{r}')>$$

$$\cdot\, e^{-2jk_o \sin\theta_i (x-x')} e^{jq(\xi-\xi')} e^{p(\xi+\xi')} \left\{ A_{y1} A_{y1}^*\, e^{-jk_z'\xi} e^{jk_z'^*\xi'} \right.$$

$$\left. + A_{y1} A_{y2}^* e^{-jk_z'\xi} e^{-jk_z'^*\xi'} + A_{y2} A_{y1}^* e^{jk_z'\xi} e^{jk_z'^*\xi'} + A_{y2} A_{y2}^* e^{jk_z'\xi} e^{-jk_z'^*\xi'} \right\} \qquad (121)$$

The term $<\mu(\underline{r})\mu(\underline{r}')>$ which appears in the above integral, was defined earlier as the correlation function, $B(\underline{r}-\underline{r}')$. In our case, we shall let $B(\underline{r}-\underline{r}')$ be isotropic, that is $B(\underline{r}-\underline{r}') = B(\,|\underline{r}-\underline{r}'|\,)$. Also, in order to make the mathematics manageable, we will assume that the form of the correlation function is exponential.

$$B(R) = e^{-R/\ell} \qquad R = |\underline{r} - \underline{r}'| \qquad (122)$$

where ℓ is the correlation distance. These assumptions about the form of the correlation function cannot be justified analytically, since no one seems to really know the nature of the correlation function for actual vegetation. The assumptions are made merely to simplify calculations so that an analytical result can be obtained quickly; then the theory can be compared to actual experimental results. The assumption of using the isotropic correlation function given above can only be justified on the basis of comparing the final result with experimental data. Also, any use of an anisotropic correlation function will result in very great mathematical complexities. The following change of variables will be made to facilitate carrying out the integration:

$$x - x' = \eta_x$$

$$y - y' = \eta_y$$

This change of variables will allow $< E_{HH} E^*_{HH} >$ to be written as follows:

$$<E_{HH} E^*_{HH}> = \frac{k_o^2 \cos^2\theta_i}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} d\eta_x \int_{-\infty}^{\infty} d\eta_y \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{o} d\xi \int_{-\infty}^{o} d\xi' \, B(\eta_x, \eta_y, \xi-\xi') \, x$$

$$e^{-2jk_o \eta_x \sin\theta_i} e^{jq(\xi-\xi')} e^{p(\xi+\xi')} \{ A_{y1} A^*_{y1} e^{-jk_z' \xi} e^{jk_z'^* \xi'} + A_{y1} A^*_{y2} e^{-jk_z' \xi} e^{-jk_z'^* \xi'}$$

$$+ A_{y2} A^*_{y1} e^{jk_z' \xi} e^{jk_z'^* \xi'} + A_{y2} A^*_{y2} e^{jk_z' \xi} e^{-jk_z'^* \xi'} \} \tag{123}$$

The integrals in x' and y' appear to be meaningless because the limits extend from minus infinity to plus infinity. However, these integrals actually form the illuminated area, since we want to have the average backscattered power equal to zero outside the illuminated area. We can now write equation (123) in the following form:

$$<E_{HH} E^*_{HH}> = \lim_{h \to \infty} \frac{(k_o^2 \cos^2\theta_i) A_o}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} d\eta_x \int_{-\infty}^{\infty} d\eta_y \int_{-h}^{o} d\xi \int_{-h}^{o} d\xi' \, B(\eta_x, \eta_y, \xi-\xi') e^{-2jk_o \eta_x \sin\theta_i}$$

$$e^{jq(\xi-\xi')} e^{p(\xi+\xi')} \{ A_{y1} A^*_{y1} e^{-jk_z' \xi} e^{jk_z'^* \xi'} + A_{y1} A^*_{y2} e^{-jk_z' \xi} e^{-jk_z'^* \xi'}$$

$$+ A^*_{y1} A_{y2} e^{jk_z' \xi} e^{jk_z'^* \xi'} + A_{y2} A^*_{y2} e^{jk_z' \xi} e^{-jk_z'^* \xi'} \} \tag{124}$$

The integration in ξ and ξ' will be performed first by using the following change of variables:

$$\eta_z = \xi - \xi'$$

$$\eta_z' = \frac{\xi + \xi'}{2}$$

Then, the differentials $d\xi \, d\xi'$ will become

$$d\xi \, d\xi' = \frac{\partial(\xi, \xi')}{\partial(\eta_z, \eta_z')} d\eta_z \, d\eta_z'$$

where the Jacobian $\frac{\partial(\xi, \xi')}{\partial(\eta_z, \eta_z')}$ is given as follows:

$$\frac{\partial(\xi, \xi')}{\partial(\eta_z, \eta_z')} = \begin{vmatrix} \frac{\partial \xi}{\partial \eta_z} & \frac{\partial \xi'}{\partial \eta_z} \\ \frac{\partial \xi}{\partial \eta'_z} & \frac{\partial \xi'}{\partial \eta'_z} \end{vmatrix} = 1$$

Figure 5 shows the area of integration for each of the two sets of variables.

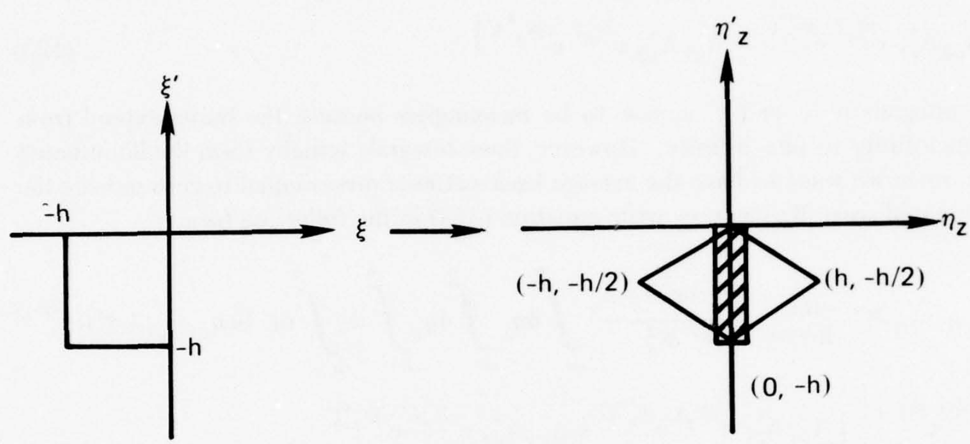

Figure 5. Areas of Integration

If we now represent the entire integrand of (124) by $f(\xi, \xi')$, then the integrals in ξ and ξ' will become

$$\lim_{h\to\infty} \int_{-h}^{0} d\xi \int_{-h}^{0} d\xi' \, f(\xi, \xi') \rightarrow \lim_{h\to\infty} \int_{-h}^{0} d\eta'_z \int_{\eta_{z1}(\eta'_z)}^{\eta_{z2}(\eta'_z)} d\eta_z \, f(\eta'_z + \tfrac{1}{2}\eta_z, \eta'_z - \tfrac{1}{2}\eta_z)$$

where $\eta_{z1}\ (\eta'_z)$ and $\eta_{z2}\ (\eta'_z)$ are defined below.

$\eta_{z1}\ (\eta'_z) = -2\ (\eta'_z + h)$	when	$-h \leqslant \eta'_z \leqslant -h/2$
$\eta_{z1}\ (\eta'_z) = 2\eta'_z$	when	$-h/2 \leqslant \eta'_z \leqslant 0$
$\eta_{z2}\ (\eta'_z) = 2\ (\eta'_z + h)$	when	$-h \leqslant \eta'_z \leqslant -h/2$

$$\eta_{z2}(\eta_z') = -2\eta_z' \qquad \text{when} \qquad -h/2 \leqslant \eta_z' \leqslant 0$$

The function $f(\eta_z' + \tfrac{1}{2}\eta_z, \eta_z' - \tfrac{1}{2}\eta_z)$ will fall off rapidly for the case where $|\eta_z|$ is greater than the correlation distance ℓ. Then, the integration is only over the shaded area in figure 5, which would be a few correlation lengths wide. Thus, the integration in η_z and η_z' now becomes

$$\lim_{h\to\infty} \int_{-h}^{0} d\xi \int_{-h}^{0} d\xi' \, f(\xi, \xi') \longrightarrow \int_{-\infty}^{0} d\eta_z' \int_{-\infty}^{\infty} d\eta_z \, f(\eta_z' + \tfrac{1}{2}\eta_z, \eta_z' - \tfrac{1}{2}\eta_z)$$

The above approximation of the integral should be valid for the case where $h \gg \ell$ (correlation distance). The equation for $<E_{HH} E_{HH}^*>$ as given by (124) now becomes

$$<E_{HH}E_{HH}^*> = \frac{(k_o \cos\theta_i)^2 A_o}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} d\eta_x \int_{-\infty}^{\infty} d\eta_y \int_{-\infty}^{\infty} d\eta_z \int_{-\infty}^{0} d\eta_z' \, B(\eta_x, \eta_y, \eta_z) \, e^{-2jk_o\eta_x\sin\theta_i} \cdot$$

$$e^{jq\eta_z} e^{2p\eta_z'} \Big\{ A_{y1}A_{y1}^* \, e^{jk_z'^*(\eta_z' + \frac{1}{2}\eta_z)} + A_{y1}A_{y2}^* \, e^{-jk_z'(\eta_z' + \frac{1}{2}\eta_z)} \, e^{-jk_z'^*(\eta_z' - \frac{1}{2}\eta_z)}$$

$$+ A_{y2}A_{y1}^* \, e^{jk_z'(\eta_z' + \frac{1}{2}\eta_z)} e^{jk_z'^*(\eta_z' - \frac{1}{2}\eta_z)} + A_{y2}A_{y2}^* \, e^{jk_z'(\eta_z' + \frac{1}{2}\eta_z)} \, e^{-jk_z'^*(\eta_z' - \frac{1}{2}\eta_z)} \Big\} \qquad (125)$$

The integration in η_z' is now quite simple, and when it is carried out, the following equation for $< E_{HH} E_{HH}^* >$ will result:

$$<E_{HH}E_{HH}^*> = \frac{(k_o \cos\theta_i)^2 A_o}{4\pi^2 R_o^2} \int_{-\infty}^{\infty} d\eta_x \int_{-\infty}^{\infty} d\eta_y \int_{-\infty}^{\infty} d\eta_z \, B(\eta_x, \eta_y, \eta_z) \, e^{-2jk_o\eta_x\sin\theta_i} \cdot$$

$$e^{jq\eta_z} \Big\{ A_{y1}A_{y1}^* \, c_1 \, e^{-j\eta_z(k_z' + k_z'^*)/2} + A_{y1}A_{y2}^* \, c_2 \, e^{-j\eta_z(k_z' - k_z'^*)/2}$$

$$+ A_{y2}A_{y1}^* c_3 \, e^{j\eta_z(k_z' - k_z'^*)/2} + A_{y2}A_{y2}^* \, c_4 \, e^{j\eta_z(k_z' + k_z'^*)/2} \Big\} \qquad (126)$$

where c_1, c_2, c_3 and c_4 are defined below:

$$c_1 = \frac{1}{2p - jk_z' + jk_z'^*}$$

$$c_2 = \frac{1}{2p - jk_z' - jk_z'^*}$$

$$c_3 = \frac{1}{2p + jk_z' + jk_z'^*}$$

$$c_4 = \frac{1}{2p + jk_z' - jk_z'^*}$$

The volume integral presented in (126) is best evaluated if a change is made to spherical coordinates. Figure 6 shows the geometric relationships between η_x, η_y, η_z and the spherical coordinates R, χ, ϕ. Also, the correlation function is assumed to be isotropic and exponential.

$$B(\eta_x, \eta_y, \eta_z) = e^{-\sqrt{\eta_x^2 + \eta_y^2 + \eta_z^2}/\ell} = e^{-R/\ell} = B(R)$$

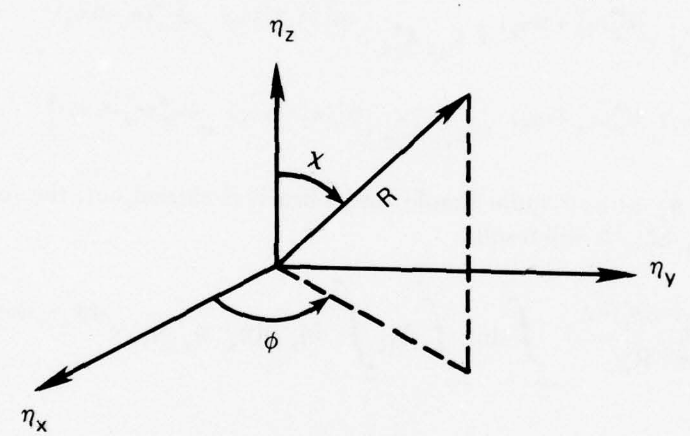

Figure 6. Spherical Coordinate Geometry

Equation (126) can now be written as follows:

$$<E_{HH} E_{HH}^*> = \frac{(k_o \cos\theta_i)^2 A_o}{4\pi^2 R_o^2} \int_o^\infty dR \int_o^\pi d\chi \int_o^{2\pi} d\phi \, R^2 \sin\chi \, e^{-R/\ell} e^{-2jk_o R \sin\chi \cos\phi \sin\theta_i} \cdot$$

$$e^{jqR\cos\chi} \Big\{ A_{y1} A_{y1}^* \, c_1 \, e^{-jR\cos\chi(k_z' + k_z'^*)/2} + A_{y1} A_{y2}^* \, c_2 \, e^{-jR\cos\chi(k_z' - k_z'^*)/2}$$

$$+ A_{y2} A_{y1}^* \, c_3 \, e^{jR\cos\chi(k_z' - k_z'^*)/2} + A_{y2} A_{y2}^* \, c_4 \, e^{jR\cos\chi(k_z' + k_z'^*)/2} \Big\} \qquad (127)$$

The integrals in (127) can now be worked out analytically and are in appendix B. After carrying out the integration, the following equation for $<E_{HH}\ E^*_{HH}>$ is obtained:

$$<E_{HH}E^*_{HH}> = \frac{2k_o^2 \ell^3 A_o \cos^2\theta_i}{\pi R_o^2} \left\{ \frac{A_{y1}A^*_{y1}c_1}{[1+b_o^2\ell^2]^2} + \frac{A_{y1}A^*_{y2}c_2}{[1+b_1^2\ell^2]^2} + \right.$$
$$\left. + \frac{A^*_{y1}A_{y2}c_3}{[1+b_2^2\ell^2]^2} + \frac{A_{y2}A^*_{y2}c_4}{[1+b_3^2\ell^2]^2} \right\} \qquad (128)$$

where the quantities b_o, b_1, b_2 and b_3 are defined below:

$$b_o = \sqrt{4k_o^2\sin^2\theta_i + [q - (k_z' + k_z'^*)/2]^2}$$

$$b_1 = \sqrt{4k_o^2\sin^2\theta_i + [q - (k_z' - k_z'^*)/2]^2}$$

$$b_2 = \sqrt{4k_o^2\sin^2\theta_i + [q + (k_z' - k_z'^*)/2]^2}$$

$$b_3 \doteq \sqrt{4k_o^2\sin^2\theta_i + [q + (k_z' + k_z'^*)/2]^2}$$

The radar backscatter coefficient can now be easily obtained from the definition provided by equation (118).

$$\sigma^o_{HH} = 8k_o^2\ell^3\cos^2\theta_i \left\{ \frac{A_{y1}A^*_{y1}c_1}{[1+b_o^2\ell^2]^2} + \frac{A_{y1}A^*_{y2}c_2}{[1+b_1^2\ell^2]^2} \right.$$
$$\left. + \frac{A_{y2}A^*_{y1}c_3}{[1+b_2^2\ell^2]^2} + \frac{A_{y2}A^*_{y2}c_4}{[1+b_3^2\ell^2]^2} \right\} \qquad (129)$$

Equation (129) is the result of all work in this section. It must be remembered that the solution assumed an isotropic exponential correlation function. Also, the effective propagation constant for the mean wave was derived using the solution of an infinite space problem. In order to be strictly correct, the effective propagation constant should be obtained from a solution of the half-space problem.

The next section of this report will be devoted to obtaining a solution for the backscatter coefficient for the case where the incident wave is vertically polarized.

Vertical Polarization Analysis. The results of the section on Scattering Geometry and Wave Propagation In a Random Medium of Infinite Extent will now be used to derive a radar backscatter coefficient for the case of an incident wave which is vertically polarized. We will now consider a wave with vertical polarization that is incident from free space ($z > 0$) onto a lossy random media ($z < 0$). The geometry of the situation is given in figure 7.

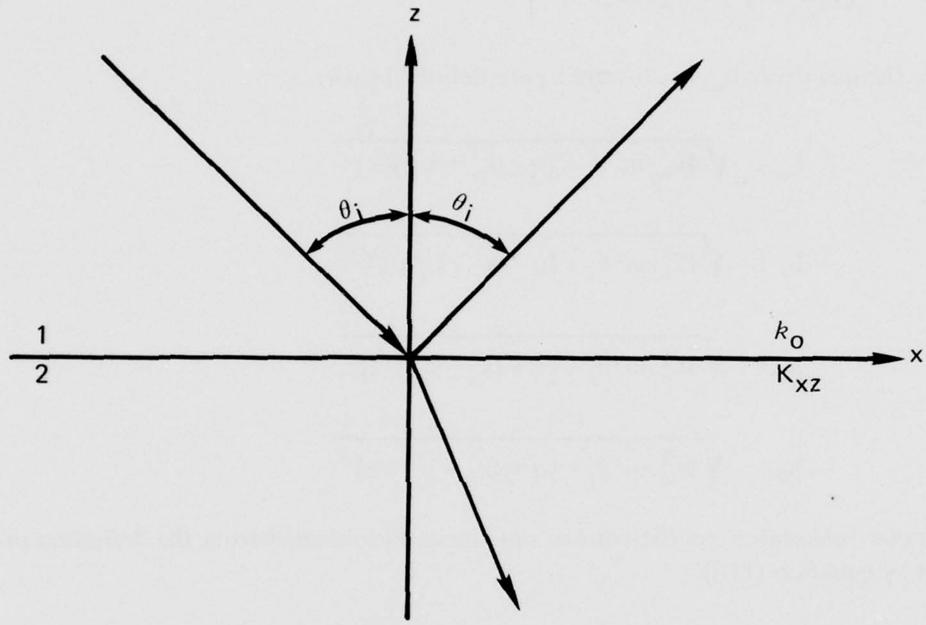

Figure 7. Geometry for the Coherent Waves (Vertical Polarization)

The polarization vector for the incident wave now lies in the xz plane. The solutions for the mean waves in both media will be obtained first; then the solution for the scattered waves will be obtained the same way as in the horizontal polarization case. The total mean magnetic field, $\underline{H}_1(\underline{r})$, in the upper medium can be written as

$$\underline{H}_1(\underline{r}) = \underline{a}_y \left[e^{-jk_o \underline{n}_i \cdot r} + R_{\parallel}\, e^{-jk_o \underline{n}_r \cdot \underline{r}} \right] \tag{130}$$

$$\underline{n}_i = \underline{a}_x \sin\theta_i - \underline{a}_z \cos\theta_i$$

$$\underline{n}_r = \underline{a}_x \sin\theta_i + \underline{a}_z \cos\theta_i$$

$$\underline{r} = x\underline{a}_x + y\,\underline{a}_y + z\,\underline{a}_z$$

$R_{\parallel}$ is a reflection coefficient.

The first term in (130) represents the incident wave, and the second term represents the reflected wave. The mean magnetic field, $\underline{H}_2(\underline{r})$, in the lower medium can be written as

$$\underline{H}_2(\underline{r}) = \underline{a}_y T_{\parallel} e^{-jK_{xz}\underline{n}_3 \cdot \underline{r}} \tag{131}$$

$$\underline{n}_3 = \underline{a}_x \sin\psi - \underline{a}_z \cos\psi$$

$T_{\parallel}$ is a transmission coefficient.
ψ is the complex angle of refraction.

Writing Snell's law, we have

$$K_{xz} \sin\psi = k_o \sin\theta_i \tag{132}$$

Again, by following Stratton[23], an equation can be derived for $\cos\psi$. If we write K_{xz} as $\beta_{xz} - j\,\alpha_{xz}$, then $\sin\psi$ is

$$\sin\psi = (\hat{a} + j\hat{b}) \sin\theta_i \tag{133}$$

where

$$\hat{a} = \beta_{xz} k_o/(\beta^2_{xz} + \alpha^2_{xz})$$

$$\hat{b} = \alpha_{xz} k_o/(\beta^2_{xz} + \alpha^2_{xz})$$

Solving now for $\cos\psi$, we have

$$\cos\psi = \sqrt{1 - (\hat{a} + j\hat{b})^2 \sin^2\theta_i} = \hat{\rho}_t e^{-j\hat{\phi}_t} \tag{134}$$

The magnitude $\hat{\rho}_t$ and the phase $\hat{\phi}_t$ can be found by squaring (134) and equating real and imaginary parts on either side of the equation.

$$\hat{\rho}_t^2 \cos 2\hat{\phi}_t = 1 + (\hat{b}^2 - \hat{a}^2) \sin^2\theta_i \tag{135}$$

$$\hat{\rho}_t^2 \sin 2\hat{\phi}_t = 2\hat{a}\hat{b} \sin^2\theta_i \tag{136}$$

Solving equations (135) and (136) for $\hat{\rho}_t$ and $\hat{\phi}_t$, we have

$$\hat{\rho}_t = \left\{ [1 - (\hat{a}^2 - \hat{b}^2)\sin^2\theta_i]^2 + 4\hat{a}^2\hat{b}^2 \sin^4\theta_i \right\}^{1/4} \tag{137}$$

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[23] Julius Adams Stratton, *Electromagnetic Theory*, McGraw Hill, New York, 1941, page 501.

$$\hat{\phi}_t = \tfrac{1}{2} \tan^{-1} \left[\frac{2\hat{a}\hat{b}\sin^2\theta_i}{1 - (\hat{a}^2 - \hat{b}^2)\sin^2\theta_i} \right] \tag{138}$$

An equation can now be written for $K_{xz} \cos\psi$ as follows:

$$K_{xz} \cos\psi = (\beta_{xz} - j\alpha_{xz})\, \hat{\rho}_t\, (\cos\hat{\phi}_t - j\sin\hat{\phi}_t)$$

$$K_{xz} \cos\psi = \hat{q} - j\hat{p} \tag{139}$$

where $$\hat{p} = \hat{\rho}_t\, (\alpha_{xz}\cos\hat{\phi}_t + \beta_{xz}\sin\hat{\phi}_t) \tag{140}$$

and $$\hat{q} = \hat{\rho}_t\, (\beta_{xz}\cos\hat{\phi}_t - \alpha_{xz}\sin\hat{\phi}_t) \tag{141}$$

The mean magnetic field in the random media can now be written as

$$\underline{H}_2(\underline{r}) = \underline{a}_y\, T_{11}\, e^{-jk_o x \sin\theta_i}\, e^{j\hat{q}z}\, e^{\hat{p}z} \tag{142}$$

From the Maxwell equation $\nabla \times \underline{H} = \underline{J} + \dfrac{\partial \underline{D}}{\partial t}$, an expression can be obtained for the mean electric field, $\underline{E}_2(\underline{r})$, in the random media.

$$\underline{E}_2(\underline{r}) = -\frac{\omega\mu_o T_{11}}{K_{xz}} \left[\underline{a}_z \left\{ \frac{k_o \sin\theta_i}{K_{xz}} \right\} + \underline{a}_x\, \hat{\rho}_t\, e^{-j\hat{\phi}_t} \right] e^{-jk_o x \sin\theta_i}\, e^{\hat{p}z}\, e^{j\hat{q}z} \tag{143}$$

The transmission and reflection coefficients can be computed from the tangential components of the electric and magnetic fields, which must be continuous across the boundary at z = 0.

$$R_{11} = \frac{K_{xz}\cos\theta_i - k_o\cos\psi}{K_{xz}\cos\theta_i + k_o\cos\psi} \tag{144}$$

$$T_{11} = \frac{2K_{xz}\cos\theta_i}{k_o\cos\psi + K_{xz}\cos\theta_i} \tag{145}$$

The mean electric field as given by (143) is now completely determined and can be used to calculate the scattered field in the lower medium.

$$\mathcal{L}\underline{E}_s = \xi < \underline{E} >$$

$$[\nabla(\nabla\cdot \underline{E}_s) - \nabla^2 \underline{E}_s - k^2 \underline{E}_s] = -\frac{k_o^2\, \epsilon_s\, \mu(\underline{r})\, \omega\mu_o\, T_{\parallel}}{K_{xz}} \left[\underline{a}_z \left(\frac{k_o \sin\theta_i}{K_{xz}} \right) + \right.$$

$$\left. \underline{a}_x\, \hat{\rho}_t\, e^{-j\hat{\phi}_t} \right] e^{-jk_o x \sin\theta_i}\, e^{\hat{p}z}\, e^{j\hat{q}z} \tag{146}$$

We shall neglect the gradient term in (146) for the same reason it was neglected for the horizontal polarization solution. The three components of (146) then become

$$\nabla^2 E_{sx} + k^2 E_{sx} = \frac{k_o^2\, \epsilon_s\, \mu(\underline{r})\, \omega\mu_o\, T_{\parallel}}{K_{xz}}\, \rho_t\, e^{-j\hat{\phi}_t}\, e^{-jk_o x \sin\theta_i}\, e^{\hat{p}z}\, e^{j\hat{q}z} \tag{147}$$

$$\nabla^2 E_{sy} + k^2 E_{sy} = 0 \tag{148}$$

$$\nabla^2 E_{sz} + k^2 E_{sz} = \frac{k_o^3\, \epsilon_s\, \mu(\underline{r})\, \omega\mu_o\, T_{\parallel}}{K_{xz}^2}\, \sin\theta_i\, e^{-jk_o x \sin\theta_i}\, e^{\hat{p}z}\, e^{j\hat{q}z} \tag{149}$$

A solution for E_{sy} can be written in the form of a Fourier transform.

$$E_{sy}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B'_y\, (k_x, k_y)\, e^{jk'_z z}\, e^{jk_x x}\, e^{jk_y y}\, dk_x dk_y \tag{150}$$

In the above equation B'_y is a function of the Fourier variables k_x and k_y. The quantity k'_z has the same definition that it had earlier, as given by equation (70). The solutions for E_{sx} and E_{sz} are not as straightforward as E_{sy} because of the terms on the right-hand side of (147) and (149). These two terms contain $\mu(\underline{r})$, which is a random function of all three coordinates. By considering E_{sx} first, a general solution can be written as

$$E_{sx}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B'_x\, (k_x, k_y, z)\, \exp\,(jk_x x + jk_y y)\, dk_x dk_y \tag{151}$$

Placing equation (151) into (147), we have

$$\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ -k_x^2\, B'_x - k_y^2\, B'_x + \frac{d^2 B'_x}{dz^2} + k^2\, B'_x \right\} \exp\,(jk_x x + jk_y y)\, dk_x dk_y$$

$$= \frac{k_o^2\, \epsilon_s\, \mu(\underline{r})\, \omega\mu_o\, T_{\parallel}}{K_{xz}}\, \hat{\rho}_t\, e^{-j\hat{\phi}_t}\, e^{-jk_o x \sin\theta_i}\, e^{j\hat{q}z}\, e^{\hat{p}z} \tag{152}$$

If we recall the definition of S (k_x, k_y, z) as given by equation (77), the above expression can be written as

$$\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{-k_x^2 B_x' - k_y^2 B_x' + \frac{d^2 B_x'}{dz^2} + k^2 B_x' - M_o S(k_x, k_y, z) e^{\hat{p}z} e^{j\hat{q}z}\right\} x$$

$$\exp(jk_x x + jk_y y)\, dk_x dk_y = 0 \tag{153}$$

$$\text{where } M_o = \frac{k_o^2 \epsilon_s \omega\mu_o T_{||} \hat{\rho}_t e^{-j\hat{\phi}_t}}{K_{xz}}$$

The integral on the left-hand side of (153) can be satisfied by setting the integrand equal to zero. When this is done, the following second order differential equation results:

$$\frac{d^2 B_x'}{dz^2} + (k^2 - k_x^2 - k_y^2) B_x' = M_o S(k_x, k_y, z) e^{j\hat{q}z} e^{\hat{p}z} \tag{154}$$

The above differential equation has exactly the same form as equation (80), which was solved previously in connection with the horizontal polarization case. Applying the same variation of parameters method to (154) will yield the following solution for $E_{sx}(x, y, z)$:

$$E_{sx}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{B_1' e^{jk_z' z} + \frac{M_o}{2jk_z'}\left[\int_{-\infty}^{z} S(k_x, k_y, \xi) e^{\hat{p}\xi} e^{j\hat{q}\xi} e^{-jk_z'\xi} d\xi\right] \cdot\right.$$

$$\left. e^{jk_z' z} - \frac{M_o}{2jk_z'}\left[\int_{-\infty}^{z} S(k_x, k_y, \xi) e^{\hat{p}\xi} e^{j\hat{q}\xi} e^{jk_z'\xi}\right] e^{-jk_z' z}\right\} \cdot$$

$$\exp(jk_x x + jk_y y)\, dk_x dk_y \tag{155}$$

where B_1' is a function of k_x and k_y. Since equation (149) for E_{sz} has exactly the same form as (147), a solution for E_{sz} can be written as

$$E_{sz}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{B_2' e^{jk_z' z} + \frac{N_o}{2jk_z'}\left[\int_{-\infty}^{z} S(k_x, k_y, \xi) e^{\hat{p}\xi} e^{j\hat{q}\xi} e^{-jk_z'\xi} d\xi\right] \cdot\right.$$

$$\left. e^{jk_z' z} - \frac{N_o}{2jk_z'}\left[\int_{-\infty}^{z} S(k_x, k_y, \xi) e^{\hat{p}\xi} e^{j\hat{q}\xi} e^{jk_z'\xi}\right] e^{-jk_z' z}\right\} \tag{156}$$

where $$N_o = \frac{k_o^3 \epsilon_s \omega\mu_o T_{\parallel} \sin\theta_i}{K_{xz}^2}$$

The components of the scattered electric field in the upper medium ($z > 0$) can be written in the same manner as was done for horizontal polarization (e.g. equations 90 to 92).

$$E'_{sx}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_x(k_x, k_y) \exp(jk_x x + jk_y y - jk_z z)\, dk_x dk_y \tag{157}$$

$$E'_{sy}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_y(k_x, k_y) \exp(jk_x x + jk_y y - jk_z z)\, dk_x dk_y \tag{158}$$

$$E'_{sz}(x,y,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_z(k_x, k_y) \exp(jk_x x + jk_y y - jk_z z)\, dk_x dk_y \tag{159}$$

where $$k_z = \sqrt{k_o^2 - k_x^2 - k_y^2}$$

Again we have six unknowns, C_x, C_y, C_z, B'_x, B'_1 and B'_2. The three unknowns of interest are C_x, C_y and C_z, which determine the scattered electric fields in the upper medium. The boundary conditions at $z = 0$ that will permit the calculation of the wanted quantities are as follows:

$$E_{sx}(x,y,0) = E'_{sx}(x,y,0) \tag{160}$$

$$E_{sy}(x,y,0) = E'_{sy}(x,y,0) \tag{161}$$

$$\frac{\partial E_{sz}}{\partial y} - \frac{\partial E_{sy}}{\partial z} = \frac{\partial E'_{sz}}{\partial y} - \frac{\partial E'_{sy}}{\partial z} \quad \text{at } z = 0 \tag{162}$$

$$\frac{\partial E_{sx}}{\partial z} - \frac{\partial E_{sz}}{\partial x} = \frac{\partial E'_{sx}}{\partial z} - \frac{\partial E'_{sz}}{\partial x} \quad \text{at } z = 0 \tag{163}$$

In addition to the four boundary conditions given above, we have the two divergence conditions that also can be used. When the appropriate equations for the scattered electric fields are substituted into the above boundary conditions along with the divergence relations, the result will yield six algebraic equations and six unknowns. The six algebraic equations are obtained in exactly the same manner as they were in the hori-

zontal polarization solution and are given below.

$$k_x C_x + k_y C_y = k_z C_z \tag{164}$$

$$k_x B_1' + k_y B_y' + k'_z B_2' = 0 \tag{165}$$

$$B_1' + \frac{M_o}{2jk_z'} (I_1 - I_2) = C_x \tag{166}$$

$$B_y' = C_y \tag{167}$$

$$k_y C_z + k_z C_y - k_y B_2' + k_z' B_y' = \frac{k_y N_o}{2jk_z'} (I_1 - I_2) \tag{168}$$

$$k_z C_x + k_x C_z + k_z' B_1' - k_x B_2' = \frac{k_x N_o (I_1 - I_2)}{2jk_z'} - \frac{M_o (I_1 + I_2)}{2j} \tag{169}$$

where $I_1 = \int_{-\infty}^{o} S(k_x, k_y, \xi) f(\xi) e^{-jk_z'\xi} \, d\xi$

$$I_2 = \int_{-\infty}^{o} S(k_x, k_y, \xi) f(\xi) e^{jk_z'\xi} \, d\xi$$

$$f(\xi) = e^{\hat{p}\xi} e^{j\hat{q}\xi}$$

When equations (164) to (169) are solved for C_x, C_y and C_z, we have

$$C_x = C_{x1} I_1 + C_{x2} I_2 \tag{170}$$

$$C_{x1} = \frac{1}{2jk_z' F_o} \{ k_x k_z k_z' N_o F_1 + k_x^2 k_z M_o F_1 - k_y k_z k_z' N_o F_2 - k_y k_x k_z M_o F_2 \}$$

$$C_{x2} = \frac{1}{2jk_z' F_o} \{ - k_x k_z k_z' N_o F_1 - 2k_z k_z'^2 M_o F_1 - k_x^2 k_z M_o F_1 + k_y k_z k_z' N_o F_2 + k_y k_x k_z M_o F_2 \}$$

$$C_y = C_{y1} I_1 + C_{y2} I_2 \tag{171}$$

$$C_{y1} = \frac{1}{2jk_z' F_o} \{ k_y k_z k_z' N_o F_3 + k_y k_x k_z M_o F_3 - k_x k_z k_z' N_o F_2$$

$$- k_x^2 k_2 M_o F_2 \}$$

$$C_{y2} = \frac{1}{2jk_z' F_o} \{ - k_y k_z k_z' N_o F_3 - k_y k_x k_z M_o F_3 + k_x k_z k_z' N_o F_2$$

$$+ 2k_z k_z'^2 M_o F_2 + k_x^2 k_z M_o F_2 \}$$

$$C_z = C_{z1} I_1 + C_{z2} I_2 \tag{172}$$

$$C_{z1} = (k_x C_{x1} + k_y C_{y1})/k_z$$

$$C_{z2} = (k_x C_{x2} + k_y C_{y2})/k_z$$

where $F_o = (k_y^2 k_z' + k_z^2 k_z' + k_y^2 k_z + k_z'^2 k_z)\ (k_z^2 k_z' + k_x^2 k_z' + k_z'^2 k_z + k_x^2 k_z)$

$$- k_x^2 k_y^2 (k_z + k_z')^2$$

$$F_1 = k_y^2 k_z' + k_z^2 k_z' + k_y^2 k_z + k_z'^2 k_z$$

$$F_2 = k_x k_y (k_z' + k_z)$$

$$F_3 = k_z^2 k_z' + k_x^2 k_z' + k_z'^2 k_z + k_x^2 k_z$$

An equation for the far zone scattered field, $\underline{E}_{sf}$, was developed earlier from the Stratton-Chu integral and has the following form:

$$\underline{E}_{sf} = \frac{2jk_o \cos\theta_i e^{-jk_o R_o}}{4\pi R_o} \left[\underline{a}_x C_x (k_o \sin\theta_i, 0) + \underline{a}_y C_y (k_o \sin\theta_i, 0) \right.$$

$$\left. + \underline{a}_z C_z (k_o \sin\theta_i, 0) \right] \tag{173}$$

The interest is now in the x and z components of (173), which will form the vertically polarized received wave, E_{vv}. The double subscript is used here to indicate the polarization of the transmitted and received waves, respectively.

$$E_{vv} = \underline{E}_s \cdot (\underline{a}_x \cos\theta_i + \underline{a}_z \sin\theta_i)$$

$$E_{vv} = \frac{2jk_o \cos\theta_i e^{-jk_oR_o}}{4\pi R_o} \left[\cos\theta_i (A_{x1} I_1 + A_{x2} I_2) + \sin\theta_i (A_{z1} I_1 + A_{z2} I_2)\right]$$

The above equation can be manipulated easily into a form that will allow a quick solution for the backscatter coefficient, σ^o_{vv}.

$$E_{vv} = \frac{2jk_o \cos\theta_i e^{-jk_oR_o}}{4\pi R_o} [A_{v1} (k_o \sin\theta_i, 0) I_1 + A_{v2} (k_o \sin\theta_i, 0) I_2] \tag{174}$$

where $A_{v1} (k_o \sin\theta_i, 0) = A_{x1} \cos\theta_i + A_{z1} \sin\theta_i$

$A_{v2} (k_o \sin\theta_i, 0) = A_{x2} \cos\theta_i + A_{z2} \sin\theta_i$

It can be seen that the form of E_{vv} as given by (174) is the same as that of E_{HH} given by equation (117). Therefore, a final result for the backscatter coefficient σ^o_{vv} can be written directly, since it will have the same form as σ^o_{HH}.

$$\sigma^o_{vv} = \frac{4\pi R_o^2}{A_o} \frac{< E_{vv} E^*_{vv} >}{\underline{E}_i \cdot \underline{E}^*_i}$$

where $\underline{E}_i \cdot \underline{E}^*_i = (k_o / \omega \epsilon_o)^2$

$$\sigma^o_{vv} = 8\omega^2 \epsilon_o^2 \ell^3 \cos^2\theta_i \left\{ \frac{A_{v1} A^*_{v1} c_1}{[1+b_o^2 \ell^2]^2} + \frac{A_{v1} A^*_{v2} c_2}{[1+b_1^2 \ell^2]^2} + \frac{A_{v2} A^*_{v1} c_3}{[1+b_2^2 \ell^2]^2} + \frac{A_{v2} A^*_{v2} c_4}{[1+b_3^2 \ell^2]^2} \right\} \tag{175}$$

The quantities $c_1, c_2, c_3, c_4, b_o, b_1, b_2, b_3$ in (175) are defined in the same manner as they were in the Horizontal Polarization Analysis section, the difference being that p and q must be replaced by $\hat{p}$ and $\hat{q}$ respectively.

$$c_1 = \frac{1}{2\hat{p} - jk_z' + jk_z'^*}$$

$$c_2 = \frac{1}{2\hat{p} - jk_z' - jk_z'^*}$$

$$c_3 = \frac{1}{2\hat{p} + jk_z' + jk_z'^*}$$

$$c_4 = \frac{1}{2\hat{p} + jk_z' - jk_z'^*}$$

$$b_o = \sqrt{4k_o^2 \sin^2\theta_i + [\hat{q} - (k_z' + k_z'^*)/2]^2}$$

$$b_1 = \sqrt{4k_o^2 \sin^2\theta_i + [\hat{q} - (k_z' - k_z'^*)/2]^2}$$

$$b_2 = \sqrt{4k_o^2 \sin^2\theta_i + [\hat{q} + (k_z' - k_z'^*)/2]^2}$$

$$b_3 = \sqrt{4k_o^2 \sin^2\theta_i + [\hat{q} + (k_z' + k_z^*)/2]^2}$$

Equation (175) is the end result of all work in this section. The next section of the report will show some calculations of σ^o_{HH} and σ^o_{vv} and will indicate the effects of changing various parameters.

DISCUSSION OF RESULTS

This section will show some numerical calculations for the theory derived in the previous sections and will indicate the effect of various parameter changes on the back-scatter coefficient. A computer program was written for the solution of equations (129) and (175). There are five input parameters to this computer program:

1. Frequency.
2. Average relative dielectric of the random medium.
3. Average conductivity of the random medium.
4. Correlation length.
5. Standard deviation of the dielectric fluctuations of the random medium.

A copy of the computer program is given in appendix C. The output of the computer program is the backscatter coefficient in decibels as a function of the incidence angle for the two polarizations. The backscatter coefficient in decibels is related to the backscatter coefficient as follows:

$$\sigma^{o} \text{ (in decibels)} = 10 \log_{10} \sigma^{o}$$

The backscatter coefficient on the right-hand side of the above equation is computed from equations (129) and (175).

Figure 8 shows a study of $k_o \ell$ variations for horizontal polarization. In this figure, the value of $k_o \ell$ varies from 0.5 to 3.0 with no average loss ($\sigma_a = 0$). It can be seen that as $k_o \ell$ increases, the level of the curve goes down, but the shape remains the same. There is a large drop from $k_o \ell = 1$ to $k_o \ell = 2$, much larger than the change from $k_o \ell = 2$ to $k_o \ell = 3$.

Figure 9 is a study of $k_o \ell$ variations for the case where $k_o \ell$ is less than 1 and $\sigma_a = 0$. As $k_o \ell$ increases from 0.2 to 0.4, the level of the curve goes down, but the shape stays essentially the same. We see also that the separation between the curves is approximately the same.

Figure 10 presents a study of $k_o \ell$ variations for the case where σ_a has the finite value of 0.01. It can be seen that as $k_o \ell$ increases from 0.2 to 0.6 the level of the curves increases instead of decreases as they did in figure 9. The spacing between curves changes also; it decreases as $k_o \ell$ increases. When $k_o \ell$ becomes equal to 1, the level of the curve is now lower than the curve for $k_o \ell = 0.6$. Thus, it can be seen that by comparing figures 9 and 10, the introduction of average loss will greatly affect the final behavior of the curves.

Figure 11 provides curves of backscatter coefficient versus $k_o \ell$ for three different values of σ_a with the angle of incidence equal to 10 degrees. It can be seen that when $\sigma_a = 0$, the curve is monotonic and decreasing. When $\sigma_a = 0.001$ and 0.01, the curve starts at some low value, increases to a maximum value, and then falls off gradually as $k_o \ell$ increases further. This is a very radical change from the case where $\sigma_a = 0$. It is not known whether this change is valid or if it is a consequence of the assumptions used to develop the mathematical model.

Figure 12 presents curves of backscatter coefficient versus angle of incidence for five different values of the average dielectric (ϵ_a). It can be seen that as ϵ_a increases, the mean level of the curve goes down. Also, as ϵ_a increases, the curves drop off faster as the angle of incidence increases. The spacing between curves decreases as ϵ_a increases, with the largest difference occurring between the $\epsilon_a = 1$ curve and the

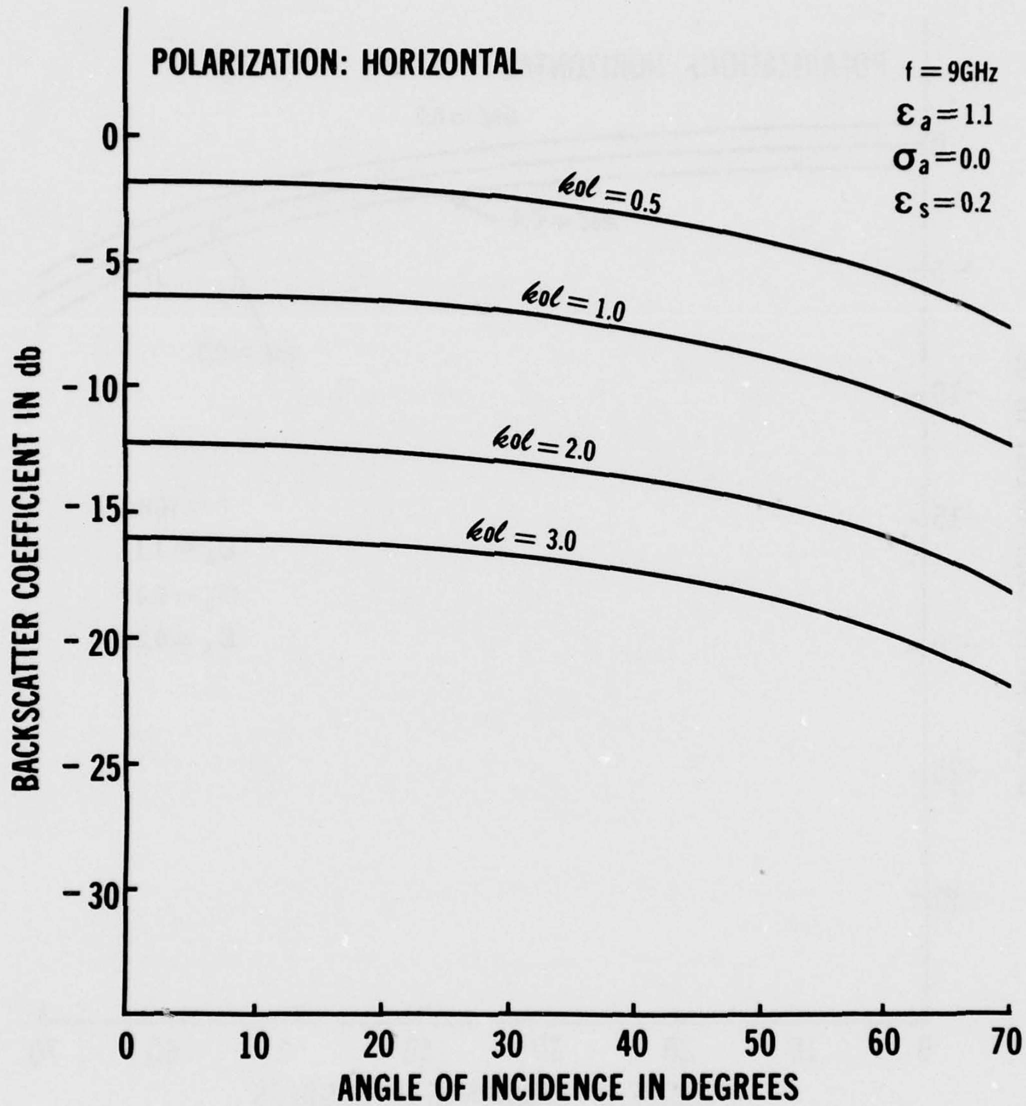

Figure 8. Study of $k_o \ell$ Variations

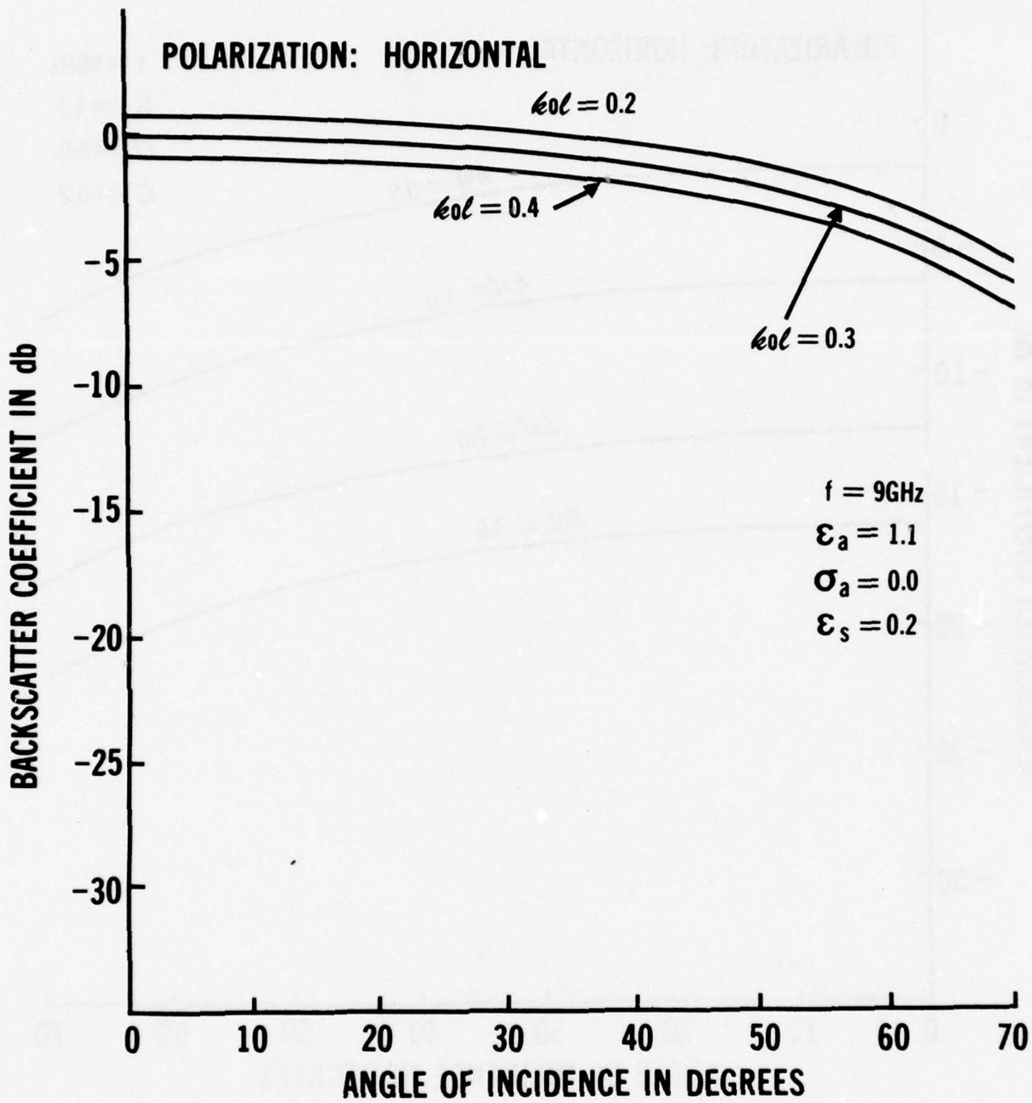

Figure 9. Study of $k_o\ell$ Variations

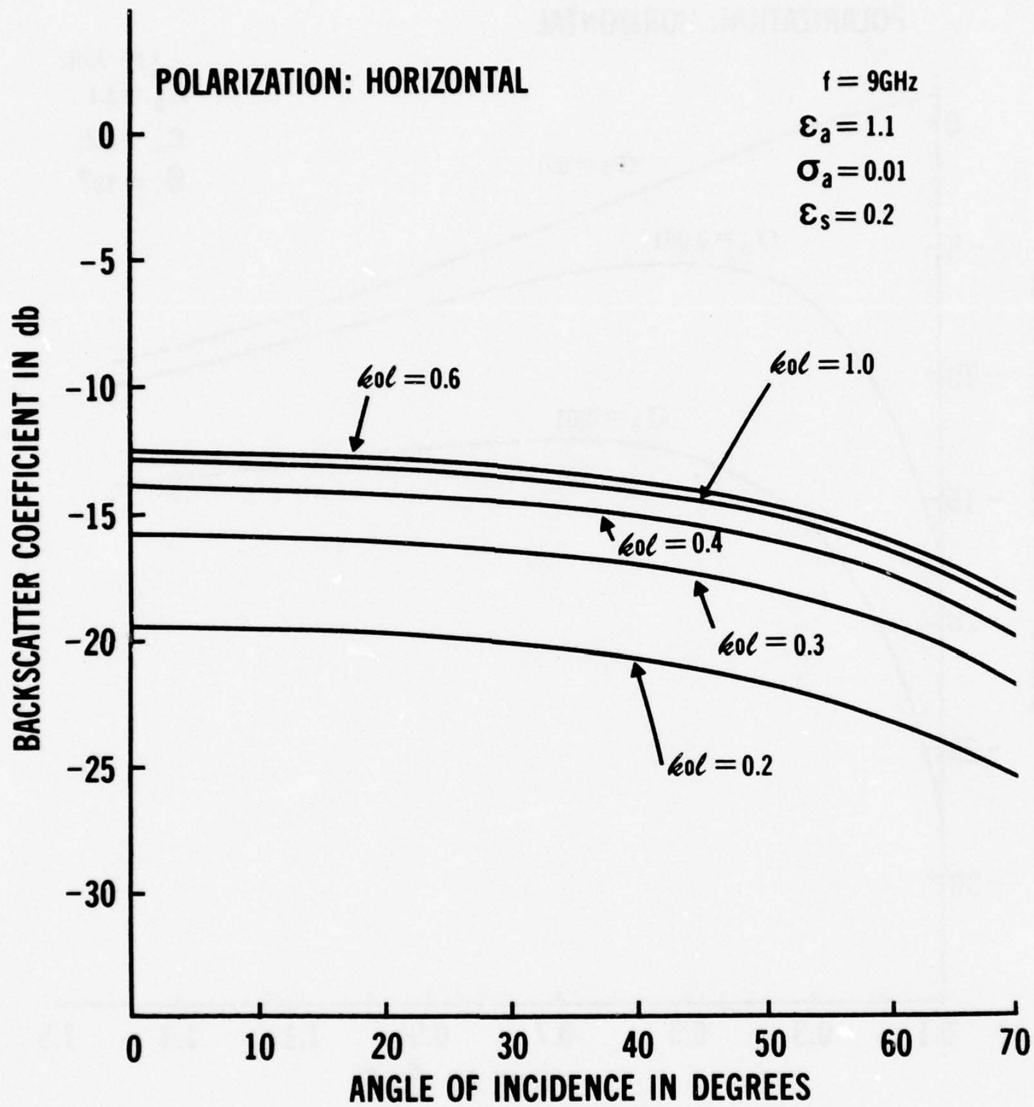

Figure 10. Study of $k_o\ell$ Variations

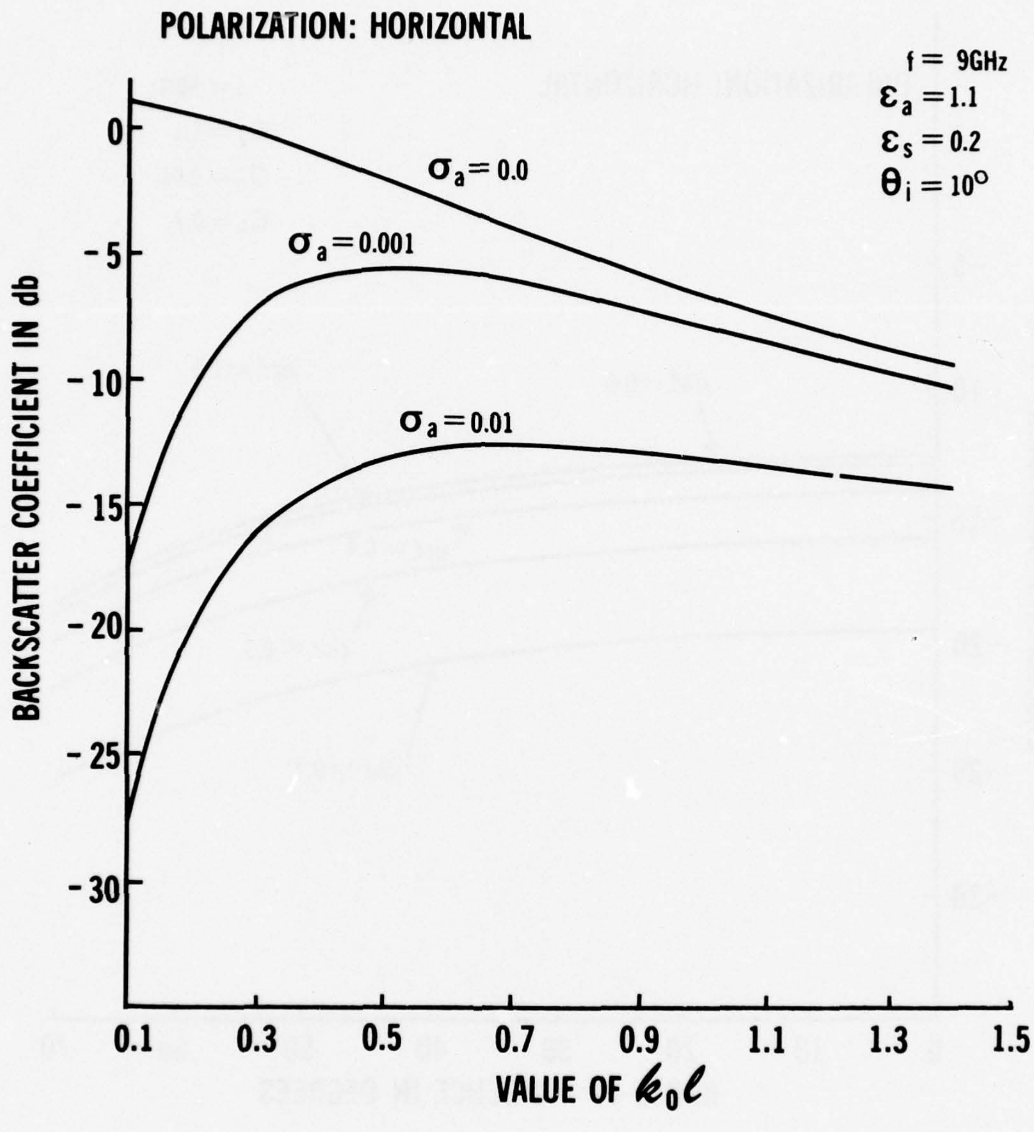

Figure 11. Study of σ_a Variations

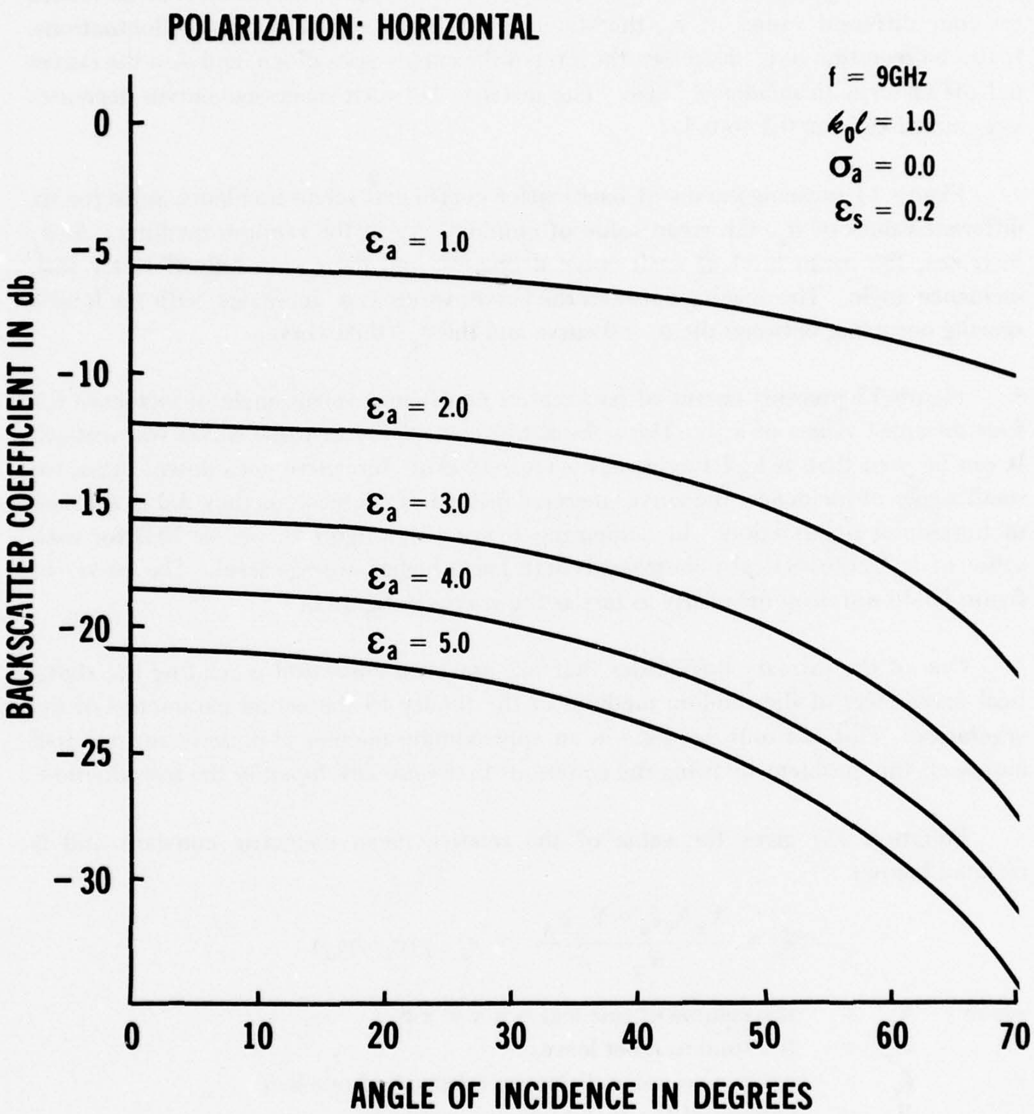

Figure 12. Study of ϵ_a Variations

$\epsilon_a = 2$ curve. When ϵ_a increases, the magnitude of the reflected coherent mean wave increases, and less energy is available for backscattering. This would appear to account for the rapid decrease in the backscatter coefficient as ϵ_a increases.

Figure 13 presents curves of backscatter coefficient versus angle of incidence for four different values of ϵ_s, the standard deviation of the dielectric fluctuations. It can be seen that as ϵ_s decreases, the level of the curves goes down, and also the curves fall off faster with incidence angle. The distance between successive curves decreases as ϵ_s increases from 0.1 to 0.4.

Figure 14 presents curves of backscatter coefficient versus incidence angle for six different values of σ_a, the mean value of conductivity in the random medium. As σ_a increases, the mean level of each curve decreases, and the curves fall off faster with incidence angle. The spacing between the curves varies as σ_a increases, with the largest spacing occurring between the $\sigma_a = 0$ curve and the $\sigma_a = 0.01$ curve.

Figure 15 presents curves of backscatter coefficient versus angle of incidence for four different values of $k_o \ell$. The polarization considered in these curves was vertical. It can be seen that as $k_o \ell$ increases, the mean level of the curve goes down. Also, for small angles of incidence, the curves increase instead of decrease, as they did in the case of horizontal polarization. In comparing figure 15 to figure 8, we see that for each value of $k_o \ell$, the vertical polarization curve has a higher average level. The curves of figure 15 do not drop off nearly as fast as the curves in figure 8.

One of the primary difficulties that has not been addressed is relating the statistical parameters of the random medium in the theory to the actual parameters of the vegetation. This can only be done in an approximate manner at present, and we shall approach this problem by using the equations that were developed in the introduction.

Equation (5) gives the value of the relative mean dielectric constant and is restated below:

$$\hat{\epsilon}_a = \frac{V_\ell N_\ell \hat{\epsilon}_\ell + V_A \epsilon_A}{V_T} = \epsilon_a - j\,(\sigma_a/\omega\epsilon_o)$$

where

V_ℓ	=	the volume of one leaf = w x w x t
N_ℓ	=	the total number leaves
$\hat{\epsilon}_\ell$	=	relative complex dielectric constant of one leaf
V_A	=	volume of the forest which is solely air
ϵ_A	=	relative dielectric constant of air = 1.0
V_T	=	the total volume of the forest = L x L x D
ω	=	radian frequency
ϵ_o	=	permittivity of free space

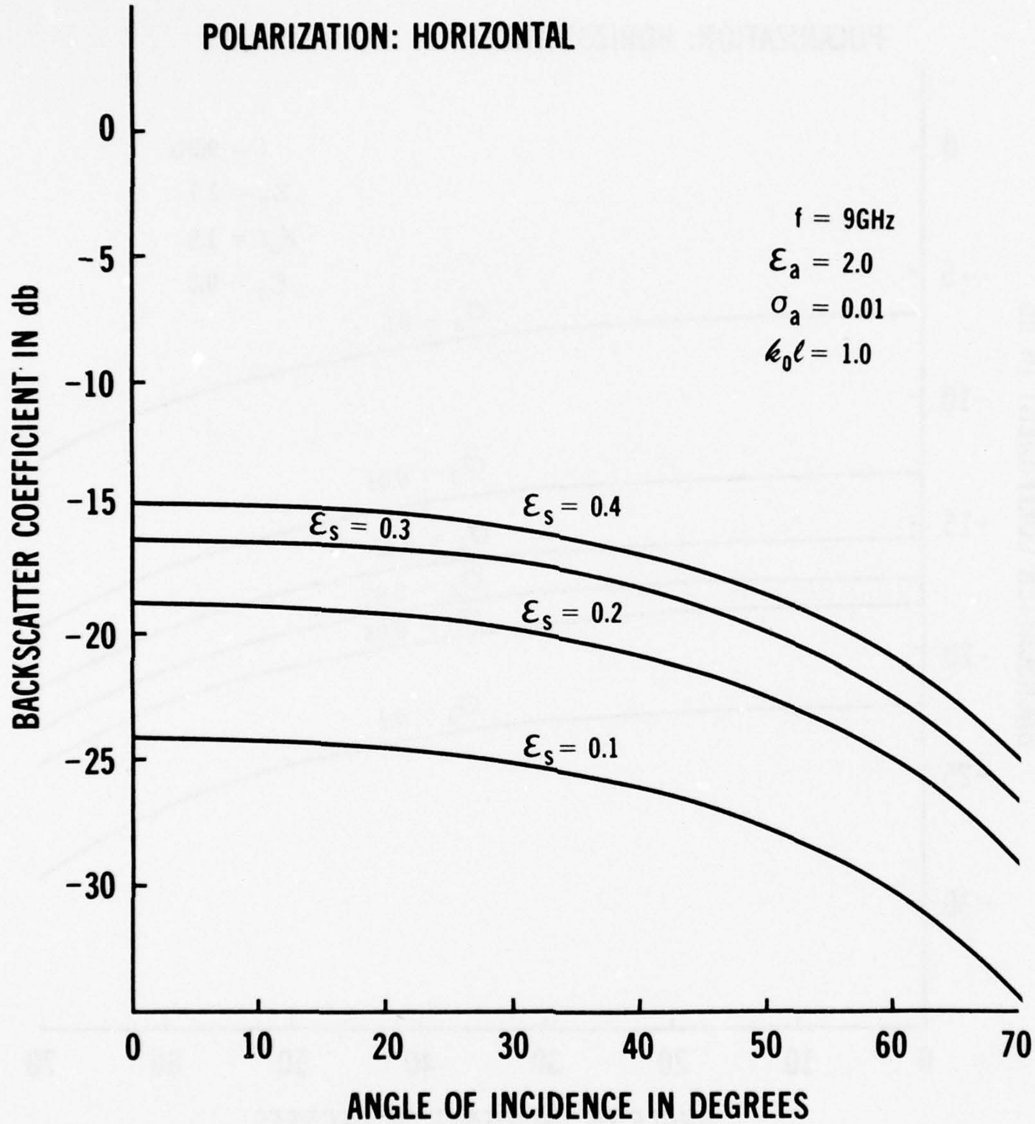

Figure 13. Study of ϵ_s Variations

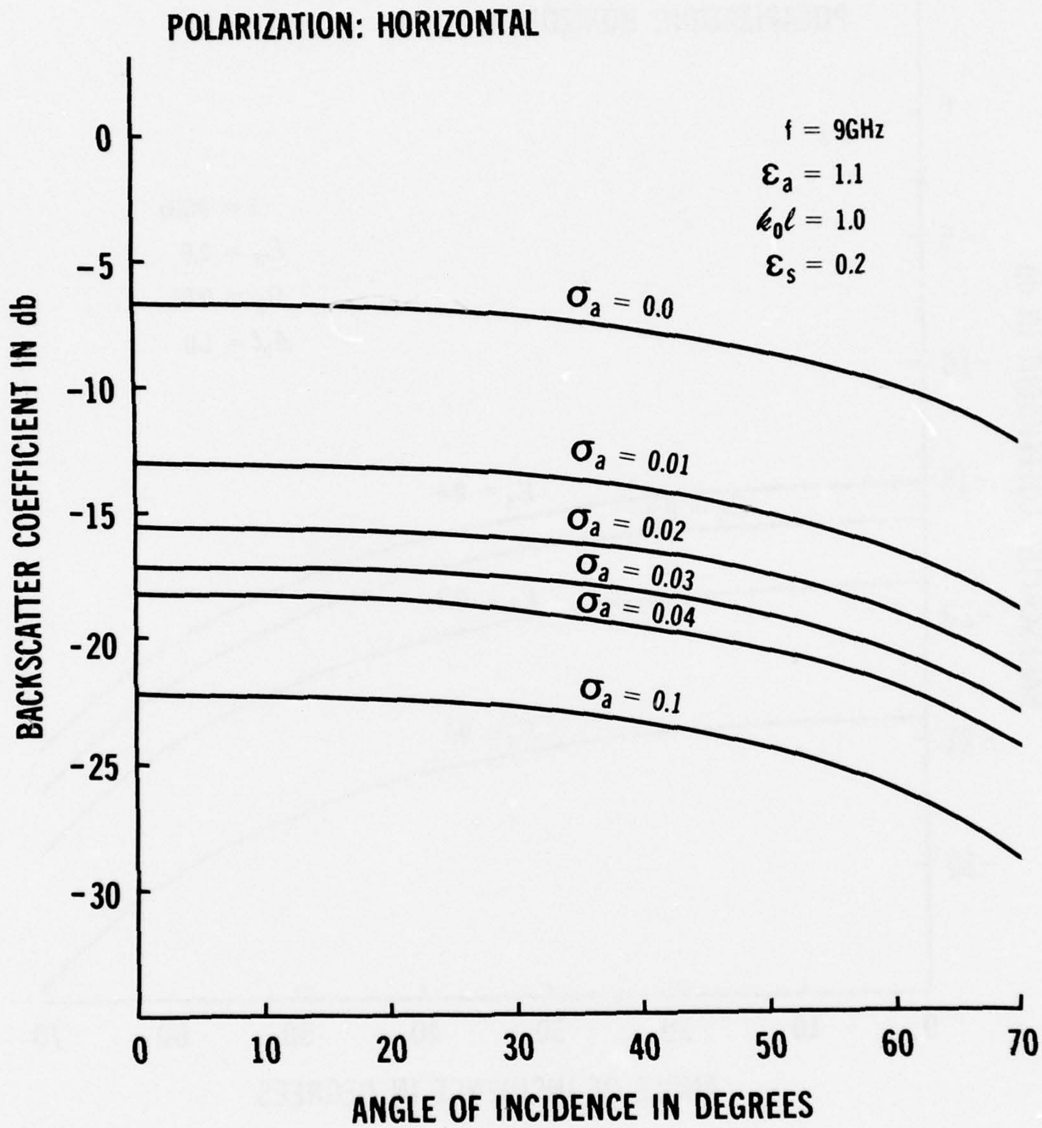

Figure 14. Study of σ_a Variations

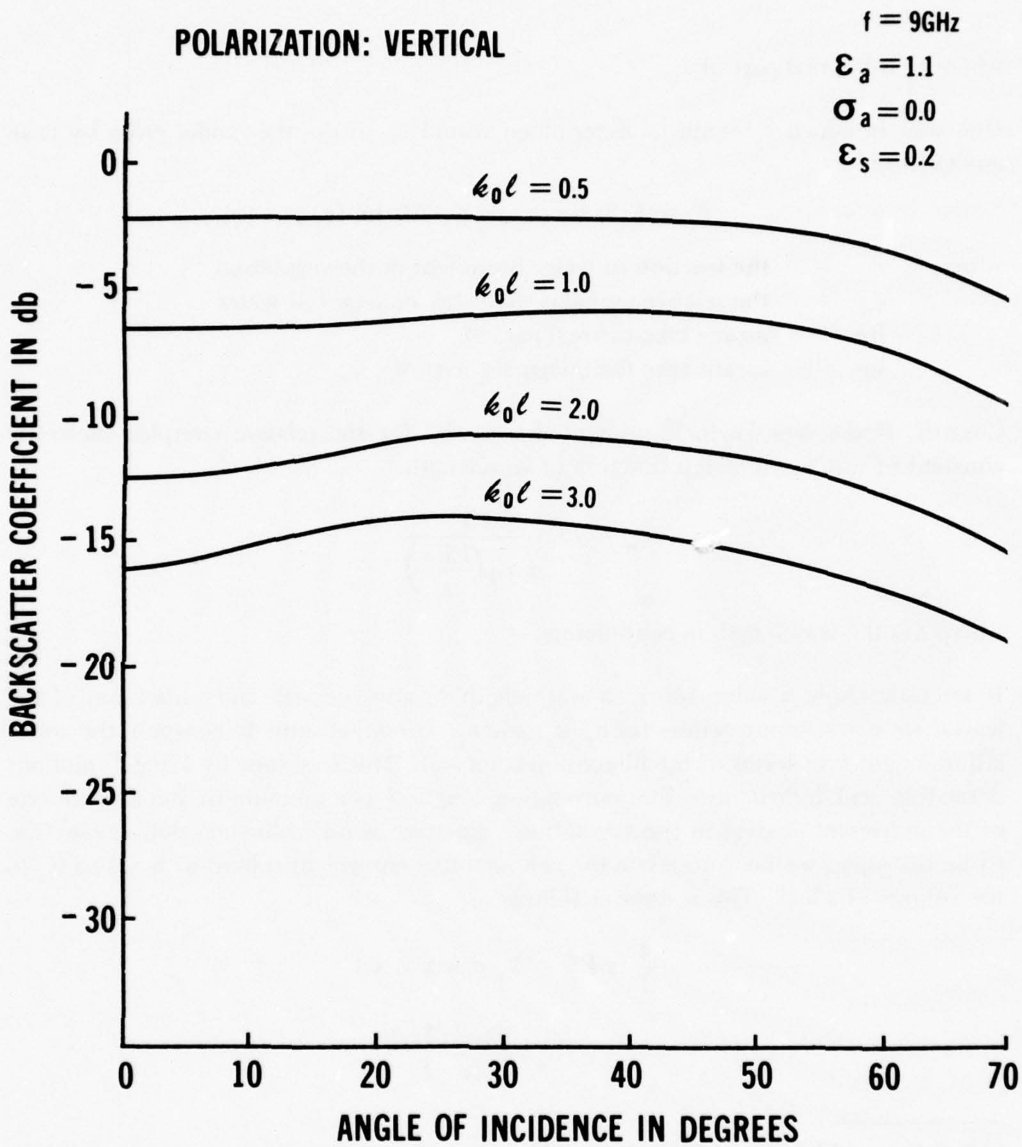

Figure 15. Study of $k_0 \ell$ Variations

The equation for computing the variance, ϵ_s^2, is as follows:

$$\epsilon_s^2 = \frac{V_\ell N_\ell (\epsilon_\ell' - \epsilon_a)^2 + V_A (\epsilon_A - \epsilon_a)^2}{V_T}$$

where ϵ_ℓ' is the real part of $\hat{\epsilon}_\ell$.

One way in which $\hat{\epsilon}_\ell$ could be determined would be to use the model given by Peak and Oliver.[24]

$$\hat{\epsilon}_\ell = (F/2)\ \mathrm{Re}\,(\epsilon_w) + j\,(F/3)\ \mathrm{Im}\,(\epsilon_w)$$

where F = the fraction of water by weight in the vegetation
ϵ_w = the relative complex dielectric constant of water
Re means take the real part of
Im means take the imaginary part of

Cosgriff, Peake, and Taylor[25] presented a model for the relative complex dielectric constant of water, which is a function of wavelength.

$$\epsilon_w = 5 + \frac{75}{1 + j\left(\frac{1.85}{\lambda}\right)}$$

where λ is the wavelength in centimeters.

If we now chose a value for F, a wavelength λ, and the size and separation of the leaves, we can compute values for ϵ_a, σ_a, and ϵ_s. It remains now to compute the correlation length ℓ in terms of the dimensions of a leaf. This was done by Fung,[26] and our derivation will follow his. The correlation length ℓ is a measure of the average size of the scatterers (leaves) in the vegetation. Since we assumed the correlation function to be isotropic, we need to relate the volume of an equivalent sphere with radius R_s to the volume of a leaf. This is done as follows:

$$\frac{4}{3}\pi R_s^3 = V_\ell = w \times w \times t$$

$$R_s = \left[\frac{3w^2 t}{4\pi}\right]^{1/3}$$

[24] W. H. Peake, and T. L. Oliver, *The Response of Terrestrial Surfaces at Microwave Frequencies,* Electro Science Laboratory, Ohio State University, Columbus, Ohio, Technical Report AFAL-TR-70-301, May 1971.

[25] R. L. Cosgriff, W. H. Peake, and R. C. Taylor, *Terrain Scattering Properties for Sensor System Design (Terrain Handbook II),* Antenna Laboratory, Ohio State University, 1959.

[26] A. K. Fung and H. S. Fung, *An Application of Scalar Renormalization to the Scattering of Electromagnetic Waves from a Three Dimensionally Inhomogeneous Medium with Strong Dielectric Fluctuations,* University of Kansas, RSL Technical Report 234-11, September 1975.

The correlation distance of a sphere can be shown to be equal to 0.63 times its radius. Therefore, an estimate of ℓ would be

$$\ell \approx 0.63\, R_s = 0.63 \left[\frac{3w^2 t}{4\pi}\right]^{1/3}$$

We can now compute the backscatter coefficient as a function of moisture content in the leaves. As a particular example, let us consider the following parameters:

L	=	the length and width of the vegetated region = 15 meters
D	=	the depth of the vegetated region = 3 meters
w	=	3 centimeters
t	=	0.05 centimeters
a	=	10 centimeters
d	=	4 centimeters
ϵ_A	=	1.0
f	=	frequency = 9 GHz

The total number of leaves in the volume can be calculated as follows:

$$N_\ell = \left[\frac{L}{w+d}\right]^2 \left[\frac{D}{a+t}\right]$$

Using the above equations, it is now a simple matter to calculate ϵ_w, $\hat{\epsilon}_\ell$, ϵ_a, σ_a, ϵ_s and ℓ for any value of F that we choose. Figure 16 presents curves of backscatter coefficient versus moisture content (F) for two different angles of incidence. It can be seen in both cases that as F increases, the backscatter coefficient also increases. However, the curves increase the fastest for values of moisture content between 10 and 30 percent. As the moisture content increases beyond 30 percent, the curves start to level off.

Figures 17 and 18 provide a comparison of the developed theory with some experimental measurements. The experimental data was taken by Ulaby,[27] using a radar spectrometer that recorded measurements from 8 to 18 GHz. The particular data used in figures 17 and 18 were taken at a frequency of 9 GHz over soybeans. The statistical parameters which are needed as input to the theory, were simply estimated because we did not know the moisture content in the leaves or the spacing of the leaves. It is not rigorous to compare theory and experiment in this manner. However, we can obtain some qualitative indications about agreement. The curve in figure 17 compares theory with experiment for horizontal polarization and with the soybeans at a height of 56 centimeters. For angles of incidence greater than 20°, we have reasonable qualitative agreement. For angles of incidence less than 20°,

[27] F. T. Ulaby, T. F. Bersh, P. P. Batlivala, and J. Cihlar, *Radar Responses to Vegetation II: 8-18 GHz Band*, University of Kansas, Lawrence, Kansas, RSL Technical Report 177-51, July 1974.

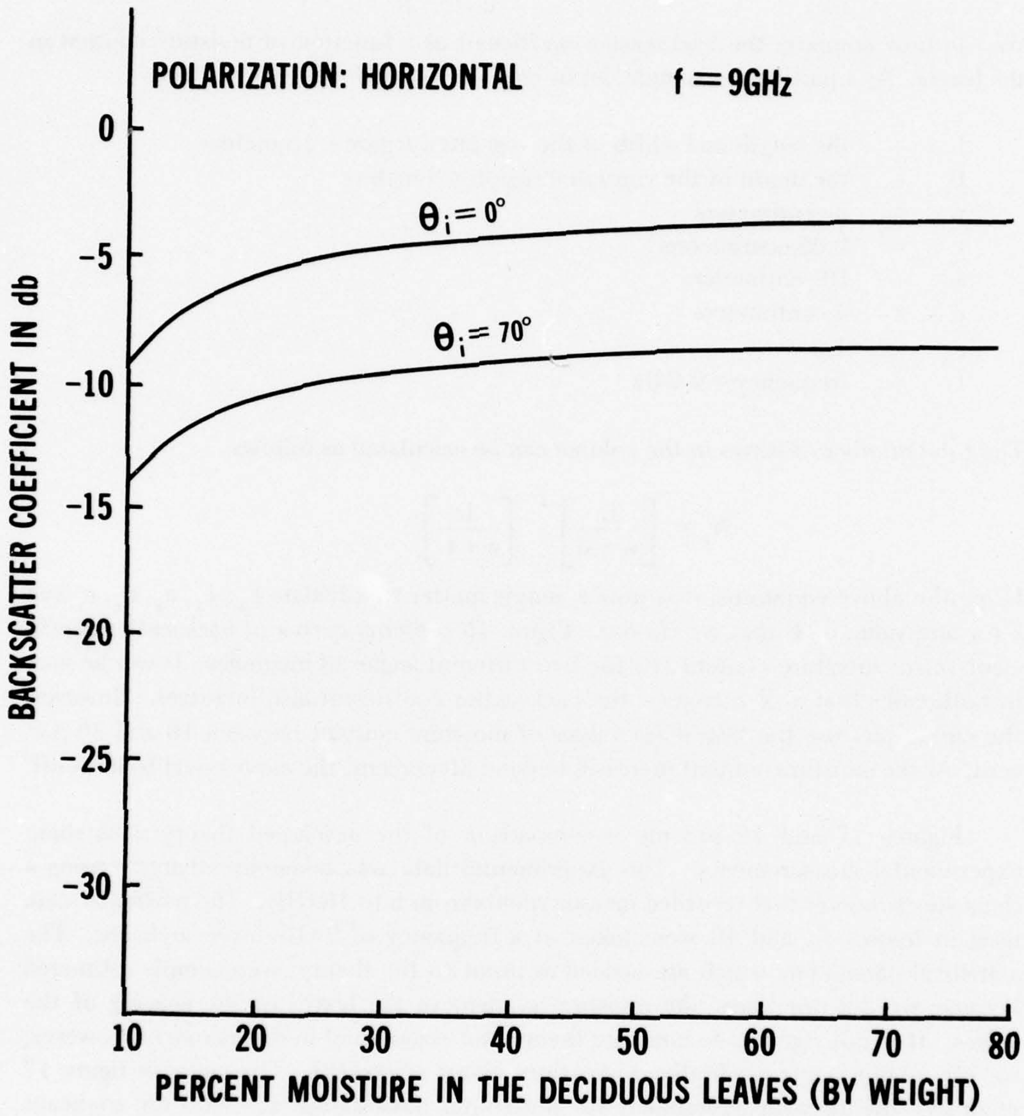

Figure 16. Study of Moisture Variations

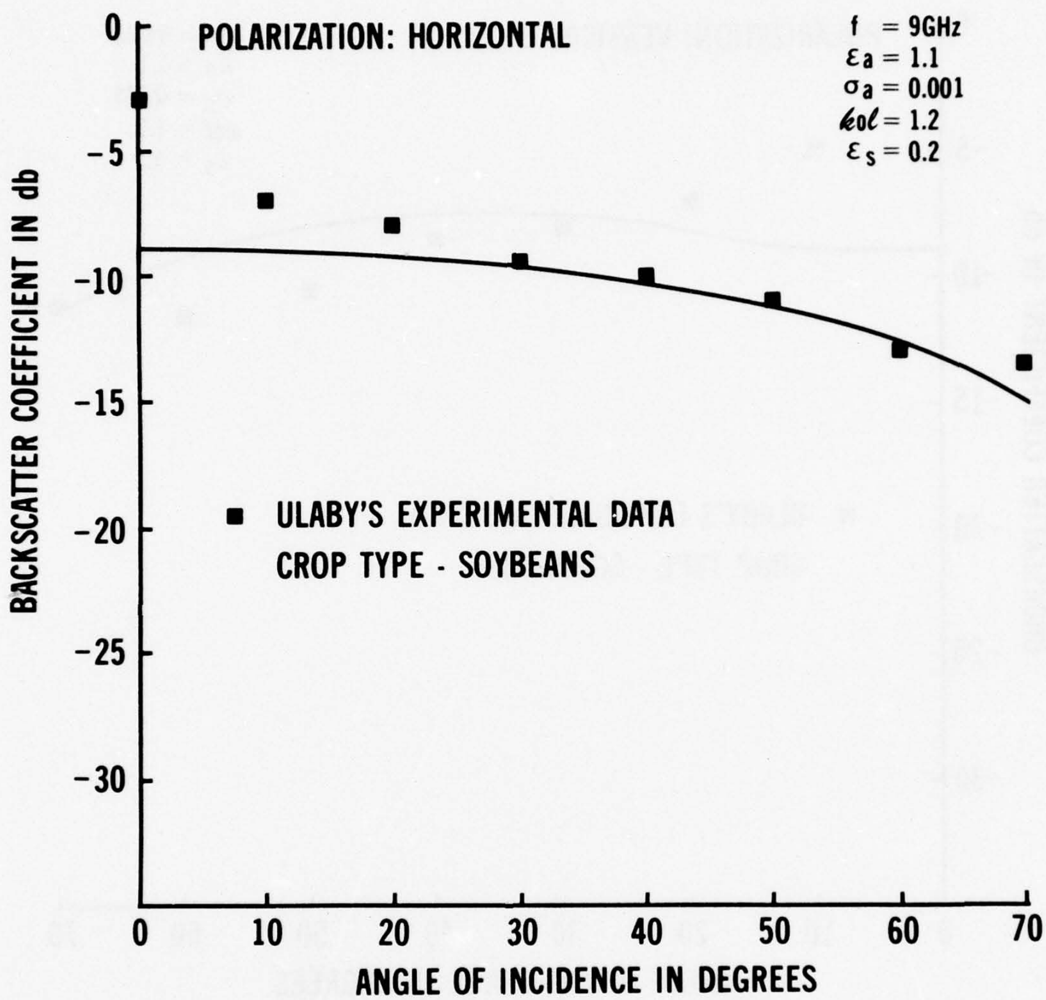

Figure 17. Comparison of Theory with Experiment for Horizontal Polarization

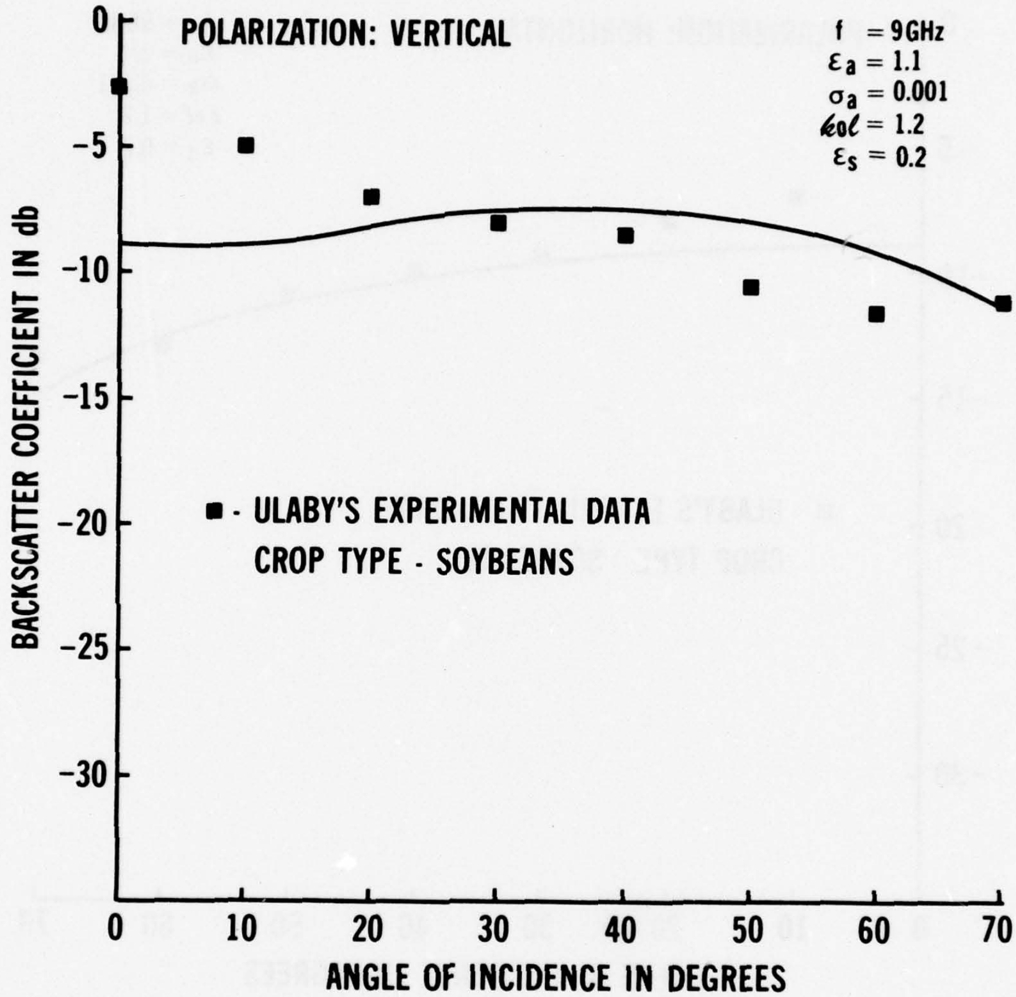

Figure 18. Comparison of Theory with Experiment for Vertical Polarization

the experimental data is considerably higher than the theory. One explanation for this is that at small angles of incidence there may be considerable ground scattering that the theoretical model does not take into account. As the angle of incidence increases, the ground effects should diminish since the ray path is almost totally in the vegetation.

We conclude this section with a brief discussion on the limitations of the theory that has been developed in this report. The idea of simulating a region of vegetation with a continuous random medium seems to be valid, but it is not known how well the renormalization technique solves the problem. One problem is that we don't know how much of the multiple scattering effect we are really considering. Another problem is that the mean wave was derived from the solution of an infinite space problem and then applied to a half-space situation. Kupiec, *et al.*,[28] have shown for the scalar problem that the mean wave in the presence of a bounded medium cannot be ascertained merely by assigning to the bounded medium the effective mean wave propagation constant for an unbounded medium. In addition, the effect of a small transition layer near the boundary must be considered. A solution for the transition layer effect for the vector problem does not exist, so its overall influence on the final solution for the backscatter coefficient is unknown.

Another problem with the present theory is that an isotropic correlation function was used to reduce the mathematical complexity; hence, it was found that no depolarization results were obtained. Although the correlation functions for real vegetation are not known, they are probably not isotropic. If anisotropic correlation functions are to be considered in the solution, the resulting mathematics may become prohibitively difficult. Also throughout our derivation, we have consistently assumed the conductivity in the random medium was a constant equal to σ_a. Obviously, this is not true since the conductivity changes significantly from air to vegetation. What is needed here is a random medium in which both the dielectric constant and the conductivity are allowed to fluctuate in a random manner. Once again the mathematics of this more realistic case will be very difficult and may even make an approximate solution elusive. Therefore, much more work needs to be done not only with the theory but also with experimental data gathering before a truly valid solution will exist for radar scattering from vegetation type areas.

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[28] I. Kupiec, *et al.*, "Reflection and Transmission by a Random Medium," *Radio Science*, Vol. 4, No. 11, November 1969, pp. 1067-1077.

CONCLUSIONS

The following conclusions can be made from the study:

a. A theory has been developed for computing radar backscatter from certain types of vegetation by using a vector renormalization approach.

b. No rigorous quantitative comparison of theory with experiment was possible; however, qualitative comparisons indicate reasonable agreement.

c. The theory developed was for like polarized (HH and VV) components only, and no analytical results were obtained for the depolarization components.

d. Although no explicit solution was obtained for the depolarization components, it was learned that one cause of depolarization is the anisotropy associated with the dielectric fluctuations of the random medium.

e. It was found that the mean wave in the vegetation attenuates for two different reasons:

1. Average loss due to water content in the vegetation.
2. Loss due to multiple scattering.

f. A simple quasi-qualitative way of relating the statistical parameters of the theory to the actual vegetation parameters was provided.

g. Using the developed theory, it was found that the backscatter coefficient increased the fastest for values of leaf moisture content between 10 and 30 percent.

APPENDIX A

CALCULATION OF THE ELEMENTS OF THE DIELECTRIC TENSOR

The purpose of this appendix is to determine the needed elements of the dielectric tensor and to obtain necessary expressions for the direction cosines of the effective propagation constant. It has already been shown that the elements ϵ_{xy} and ϵ_{yz} will be zero for an isotropic correlation function. The elements that need to be computed are ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, and ϵ_{xz}. We will consider the calculation of ϵ_{xx} in detail, and only state the results for the other elements.

$$\epsilon_{xx} = k^2/k_o^2 + k_o^2\,\epsilon_s^2 \int_{v'} B(\underline{r}-\underline{r}') \left\{ f_2(R) + \frac{(x-x')^2}{R^2}\, f_3(R) \right\} e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

$$\text{Let } I = \int_{v'} B(\underline{r}-\underline{r}') \left\{ f_2(R) + \frac{(x-x')^2}{R^2}\, f_3(R) \right\} e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}\, \underline{dr}'$$

We let $B(\underline{r} - \underline{r}') = B(|\underline{r} - \underline{r}'|) = e^{-R/\ell}$, $(R = |\underline{r} - \underline{r}'|)$, and where ℓ is the correlation distance. Also we will change the variables of integration such that $u = x\text{-}x'$, $v = y\text{-}y'$, and $w = z\text{-}z'$.

$$I = \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, B(R) \left\{ f_2(R) + \frac{U^2}{R^2}\, f_3(R) \right\} e^{j\underline{K}\cdot(\underline{r}-\underline{r}')}$$

The geometry of the $\underline{K}$ and $\underline{r} - \underline{r}'$ vectors is given below:

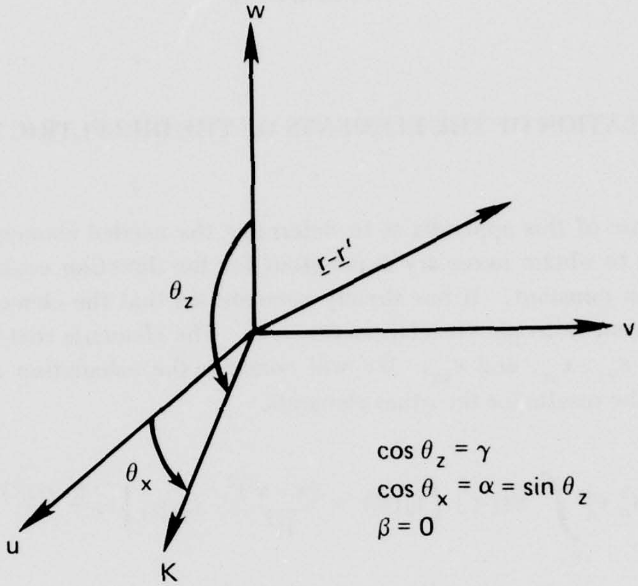

Figure A1. Geometry of the $\underline{K}$ and $\underline{r}$-$\underline{r}'$ Vectors

We will now rotate the u, w plane until the direction of the new w axis (w′) coincides with the direction of K. The geometry of the rotated coordinate system is given below.

$$u' = R\sin\theta\cos\phi$$
$$w' = R\cos\theta$$
$$v' = R\sin\theta\sin\phi$$

$$u = u'\cos\theta_z + w'\sin\theta_z$$
$$w = -u'\sin\theta_z + w'\cos\theta_z$$
$$v = v'$$

$$\left|\frac{\partial(u, v, w)}{\partial(u', v', w')}\right| = 1 \equiv \text{Jacobian}$$

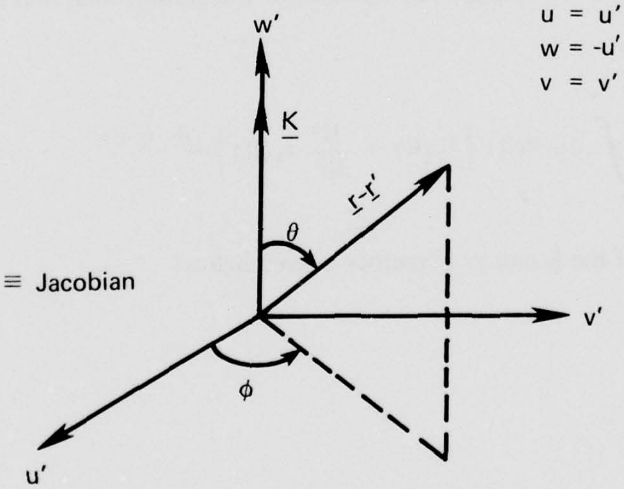

Figure A2. Geometry of the Rotated Coordinate System

The integral I now becomes

$$I = \int_{-\infty}^{\infty} du' \int_{-\infty}^{\infty} dv' \int_{-\infty}^{\infty} dw' \, B(R) \left\{ f_2(R) + \frac{(u'\gamma + w'\alpha)^2}{R^2} f_3(R) \right\} e^{j\underline{K} \cdot (\underline{r} - \underline{r}')}$$

If we change now to the spherical coordinates R, θ, and ϕ, the integral I will be:

$$I = \int_{0}^{2\pi} d\phi \int_{0}^{\pi} d\theta \int_{0}^{\infty} dR \; R^2 \sin\theta B(R) \left\{ f_2(R) + (\gamma^2 \sin^2\theta \cos^2\phi + \right.$$

$$\left. 2\alpha\gamma \cos\theta \sin\theta \cos\phi + \alpha^2 \cos^2\theta) \, f_3(R) \right\} e^{jKR\cos\theta}$$

We can break up the above complicated integral into the sum of a number of smaller integrals, the first of which to be considered is I_1.

$$I_1 = \int_{0}^{2\pi} d\phi \int_{0}^{\pi} d\theta \int_{0}^{\infty} dR \; R^2 \sin\theta B(R) \, f_2(R) \, e^{jKR\cos\theta}$$

By integrating in ϕ, we have

$$I_1 = 2\pi \int_{0}^{\pi} d\theta \int_{0}^{\infty} dR \; R^2 \sin\theta B(R) \, f_2(R) \, e^{jKR\cos\theta}$$

Carrying out the integration in θ will yield

$$I_1 = 4\pi \int_{0}^{\infty} dR \; \frac{\sin KR}{KR} \; R^2 \, B(R) \, f_2(R)$$

The second integral to be considered that is a part of I will be designated I_2 and is given as follows:

$$I_2 = \int_{0}^{2\pi} d\phi \int_{0}^{\pi} d\theta \int_{0}^{\infty} dR \, R^2 \sin^3\theta \; B(R) \; \gamma^2 \cos^2\phi \; f_3 \; (R) \; e^{jKR\cos\theta}$$

When the integration in ϕ is performed, the following result is obtained for I_2:

$$I_2 = \pi \int_0^{\pi} d\theta \int_0^{\infty} dR R^2 \sin^3\theta B(R)\, \gamma^2 f_3(R)\, e^{jKR\cos\theta}$$

The integration in θ can be easily performed by using the method of integration by parts. When this is completed, I_2 will become:

$$I_2 = \frac{4\pi\gamma^2}{K^2} \int_0^{\infty} dR\, B(R)\, f_3(R) \left[\frac{\sin(KR)}{KR} - \cos(KR) \right]$$

The third integral to be considered as a part of I will be designated I_3 and is given below as

$$I_3 = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{\infty} dR\, R^2 \sin^2\theta B(R)\, 2\alpha\gamma\cos\theta\cos\phi f_3(R)\, e^{jKR\cos\theta}$$

Integrating in ϕ from zero to 2π will give zero, and so I_3 is identically zero.

$$I_3 = 0$$

The fourth integral which forms a part of I will be designated I_4 and is as follows:

$$I_4 = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{\infty} dR\, R^2 \sin\theta B(R)\, \alpha^2 \cos^2\theta f_3(R)\, e^{jKR\cos\theta}$$

Carrying out the integration in ϕ will yield the following result for I_4:

$$I_4 = 2\pi \int_0^{\pi} d\theta \int_0^{\infty} dR\, R^2 \sin\theta B(R)\, \alpha^2 \cos^2\theta f_3(R)\, e^{jKR\cos\theta}$$

The integration in θ can be carried out in a straightforward manner to give

$$I_4 = 2\pi \int_0^{\infty} dR\, R^2 B(R)\, \alpha^2 f_3(R) \;\frac{2}{(KR)^3}\; \left[(K^2R^2-2)\sin(KR) + 2KR\cos(KR)\right]$$

Since I is equal to the sum of I_1, I_2, I_3 and I_4, we can write an equation for I involving an integration in R.

$$I = \int_0^\infty dR \left\{ \frac{4\pi \sin(KR)}{KR} R^2 f_2(R) + \frac{4\pi\gamma^2}{K^2} f_3(R) \left[\frac{\sin(KR)}{KR} - \cos(KR) \right] \right.$$

$$\left. + \frac{4\pi\alpha^2}{K^3 R} f_3(R) \left[(K^2 R^2 - 2)\sin(KR) + 2KR\cos(KR) \right] \right\} B(R)$$

Before proceeding to integrate in R, we shall first replace K with k, which is the first approximation of K when $\epsilon_s = 0$. It could be argued that K should be kept as it is in the integral I, and then the solution for ϵ_{xx} (and all the other elements of the dielectric tensor) will be a function of K. This would make the final solutions for K_{yy} and K_{xz} implicit, and some type of iterative solution would have to be found. However, the original equation for the mean wave is only good to order ϵ_s^2, so a complicated iterative solution for K_{yy} and K_{xz} would not be meaningful. It is with this argument in mind that we replace K with k in the integral.

$$I \approx \int_0^\infty dR \left\{ \frac{4\pi \sin(kR)}{kR} R^2 f_2(R) + \frac{4\pi\gamma^2}{k^2} f_3(R) \left[\frac{\sin(kR)}{kR} - \cos(kR) \right] \right.$$

$$\left. + \frac{4\pi\alpha^2}{k^3 R} f_3(R) \left[(k^2 R^2 - 2)\sin(kR) + 2kR\cos(kR) \right] \right\} B(R)$$

When the appropriate quantities for $f_2(R)$, $f_3(R)$ and $B(R)$ are substituted into the above equation for I, and after some algebraic manipulation, the following result is obtained for I:

$$I \approx \int_0^\infty dR \left\{ 4\pi k (1 - \alpha^2)\sin(kR) + 4\pi j (3\alpha^2 - 1) \frac{\sin(kR)}{R} + \right.$$

$$+ \frac{12\pi j}{kR^2} (\gamma^2 - 2\alpha^2) \left[\frac{\sin(kR)}{kR} - \cos(kR) \right] +$$

$$+ \frac{4\pi}{R} (2\alpha^2 - \gamma^2) \left[\frac{\sin(kR)}{kR} - \cos(kR) \right] + \frac{12\pi\gamma^2}{k^2 R^3} \left[- kR\sin(kR) + \right.$$

$$\left. \left. 3 \left(\frac{\sin(kR)}{kR} - \cos(kR) \right) \right] - \frac{8\pi}{k^2 R^3} \left[- kR\sin(kR) + 3 \left(\frac{\sin(kR)}{kR} - \cos(kR) \right) \right] \right\} \times$$

$$\frac{e^{-aR}}{4\pi k^2}$$

where $a = 1/\ell + jk$

By using the following integrals, an algebraic result can be obtained for I.

$$\int_0^\infty e^{-ax} \sin kx \, dx = \frac{k}{k^2 + a^2}$$

$$\int_0^\infty e^{-ax} \frac{\sin kx}{x} \, dx = \cot^{-1} \left(\frac{a}{k}\right)$$

$$\int_0^\infty e^{-ax} \frac{1}{x} \left(\frac{\sin kx}{kx} - \cos kx\right) dx = 1 - \frac{a}{k} \cot^{-1} (a/k)$$

$$\int_0^\infty e^{-ax} \frac{1}{x^2} \left(\frac{\sin kx}{kx} - \cos kx\right) dx = - \frac{k}{2} \left[\frac{a}{k} - (1 + a^2/k^2) \cot^{-1} (a/k)\right]$$

$$\int_0^\infty e^{-ax} \frac{1}{x^3} \left\{- kx \sin (kx) + 3 \left(\frac{\sin kx}{kx} - \cos kx\right)\right\} dx = \frac{k^2}{2} \left\{\frac{2}{3} + \left(\frac{a}{k}\right)^2 \right.$$

$$\left. - \frac{a}{k} \cot^{-1} \left(\frac{a}{k}\right) - \left(\frac{a}{k}\right)^3 \cot^{-1} (a/k)\right\}$$

$$I \approx \gamma^2 \left[\frac{1}{k^2 + a^2}\right] + \frac{j(3\alpha^2 - 1)}{k^2} \cot^{-1} (a/k) + \frac{j3(2\alpha^2 - \gamma^2)}{2k^2} \left[\frac{a}{k}\right.$$

$$\left. - \left(1 + \frac{a^2}{k^2}\right) \cot^{-1} \left(\frac{a}{k}\right)\right] + \frac{(2\alpha^2 - \gamma^2)}{k^2} \left[1 - \frac{a}{k} \cot^{-1} \left(\frac{a}{k}\right)\right]$$

$$+ \frac{3\gamma^2}{2k^2} \left[\frac{2}{3} + \left(\frac{a}{k}\right)^2 - \frac{a}{k} \cot^{-1} \left(\frac{a}{k}\right) - \left(\frac{a}{k}\right)^3 \cot^{-1} (a/k)\right]$$

$$- \frac{1}{k^2} \left[\frac{2}{3} + (a/k)^3 - \frac{a}{k} \cot^{-1} (a/k) - \left(\frac{a}{k}\right)^3 \cot^{-1} \left(\frac{a}{k}\right)\right]$$

An approximate analytical expression for ϵ_{xx} can now be written as follows:

$$\epsilon_{xx} \approx k^2/k_o^2 + k_o^2 \epsilon_s^2 I$$

The only difficulty in computing I comes from the arc cotangent term since both a and k are complex. The following analysis is provided to show a solution to this problem:

$$\text{Let } A = \cot^{-1}(a/k) = \cot^{-1}\left(\frac{1/\ell + jk}{k}\right)$$

Using the fact that $k = \beta_o - j\alpha_o$, we have

$$\cot^{-1}\left[\frac{1/\ell + \alpha_o + j\beta_o}{\beta_o - j\alpha_o}\right] = \cot^{-1}\left[\frac{(1/\ell + \alpha_o + j\beta_o)(\beta_o + j\alpha_o)}{\beta_o^2 + \alpha_o^2}\right]$$

Writing the equation in the form of cotA, we will have

$$\cot A = \frac{\cos A}{\sin A} = \frac{(1/\ell + \alpha_o + j\beta_o)(\beta_o + j\alpha_o)}{\beta_o^2 + \alpha_o^2} = \frac{(e^{jA} + e^{-jA})/2}{(e^{jA} - e^{-jA})/2j} = \frac{j(e^{2jA} + 1)}{(e^{2jA} - 1)}$$

$$j(e^{2jA} + 1)(\beta_o^2 + \alpha_o^2) = (1/\ell + \alpha_o + j\beta_o)(\beta_o + j\alpha_o)(e^{2jA} - 1)$$

Solving the above equation for e^{2jA} in terms of ℓ, α_o and β_o will give

$$e^{2jA} = \frac{[\beta_o^2 + 2j\beta_o\ell(\beta_o^2 + \alpha_o^2) + \alpha_o(2\beta_o^2\ell + 2\alpha_o^2\ell + \alpha_o)]}{\beta_o^2 + \alpha_o^2}$$

$$e^{2jA} = r_1 + jr_2$$

where $r_1 = 1 + 2\alpha_o\ell$

$r_2 = 2\beta_o\ell$

Changing from rectangular to polar coordinates, we will have

$$e^{2jA} = r_o e^{j\phi_o}$$

where $r_o = \sqrt{r_1^2 + r_2^2}$

$\phi_o = \tan^{-1}(r_2/r_1)$ when $r_1 > 0$

$\phi_o = \pi + \tan^{-1}(r_2/r_1)$ when $r_1 < 0$ and $r_2 > 0$

$\phi_o = \tan^{-1}(r_2/r_1) - \pi$ when $r_1 < 0$ and $r_2 < 0$

The quantity Λ can now be easily obtained.

$$\Lambda = \tfrac{1}{2}(\phi_o - j\ln r_o) = \cot^{-1}(a/k)$$

The rest of the nonzero elements of the dielectric tensor can be derived in the same manner as ϵ_{xx} was derived. Instead of deriving each of the other elements in detail, we shall only state the final results.

$$\epsilon_{yy} \approx k^2/k_o^2 + k_o^2\,\epsilon_s^2\,L$$

$$\text{where } L = \frac{1}{k^2 + a^2} - \frac{j}{k^2}\cot^{-1}(a/k) - \frac{3j}{2k^2}\left[\frac{a}{k} - \left(1 + \frac{a^2}{k^2}\right)\cot^{-1}(a/k)\right]$$

$$+ \frac{1}{2k^2}\left[2/3 + (a/k)^2 - \frac{a}{k}\cot^{-1}(a/k) - (a/k)^3\cot^{-1}(a/k)\right] +$$

$$+ \frac{1}{k^2}\left[\frac{a}{k}\cot^{-1}(a/k) - 1\right] \qquad a = 1/\ell + jk$$

$$\epsilon_{zz} \approx k^2/k_o^2 + k_o^2\,\epsilon_s^2\,P$$

$$\text{where } P = \frac{\alpha^2}{k^2 + \alpha^2} + \frac{j(3\gamma^2 - 1)}{k^2}\cot^{-1}(a/k) +$$

$$+ \frac{3j(2\gamma^2 - \alpha^2)}{2k^2}\,[a/k - (1 + a^2/k^2)\cot^{-1}(a/k)] +$$

$$+ \frac{(2\gamma^2 - \alpha^2)}{k^2}\left[1 - \frac{a}{k}\cot^{-1}(a/k)\right] + \frac{3\alpha^2}{2k^2}\left[\frac{2}{3} + (a/k)^2 - (a/k)\cot^{-1}(a/k)\right.$$

$$\left.- (a/k)^3\cot^{-1}(a/k)\right] - \frac{1}{k^2}\left[\frac{2}{3} + (a/k)^2 - \frac{a}{k}\cot^{-1}(a/k) - (a/k)^3\cot^{-1}(a/k)\right]$$

$$\epsilon_{xz} \approx k_o^2\,\epsilon_s^2\,K$$

$$\text{where} \quad K = \frac{3j\gamma\alpha}{k^2}\cot^{-1}(a/k) - \frac{\gamma\alpha}{k^2 + a^2} - \frac{3\gamma\alpha}{2k^2}\left[\frac{2}{3} + (a/k)^2\right.$$

$$- (a/k) \cot^{-1} (a/k) - (a/k)^3 \cot^{-1} (a/k) \Big] + \frac{j9\gamma\alpha}{2k^2} \Big[a/k$$

$$- (1 + a^2/k^2) \cot^{-1} (a/k) \Big] + \frac{3\gamma\alpha}{k^2} \left[1 - \frac{a}{k} \cot^{-1} (a/k) \right]$$

We will now consider an approximate solution for the direction cosines α and γ. For a vertically polarized wave, we have

$$K_{xz} \sin\psi = k_o \sin\theta_i \quad \text{where} \quad \sin\psi = \alpha$$

$$K_{xz} = \beta_{xz} - j\alpha_{xz}$$

$$\sin\psi = (\hat{a} + j\hat{b}) \sin\theta_i = \alpha$$

where $\hat{a} = \beta_{xz} k_o / (\beta_{xz}^2 + \alpha_{xz}^2)$

$\hat{b} = \alpha_{xz} k_o / (\beta_{xz}^2 + \alpha_{xz}^2)$

When the elements of the dielectric tensor and the effective propagation constant (K_{xz}) are being computed and a value for α is needed, we can simply replace K_{xz} by its first approximation which is k. This would mean that β_{xz} and α_{xz} would be replaced by β_o and α_o, respectively in the above equations for $\hat{a}$ and $\hat{b}$. Also when γ is computed, we would use the following expression:

$$\gamma = -\cos\psi = -\sqrt{1 - (\hat{a} + j\hat{b})^2 \sin^2\theta_i} = -\hat{\rho}_t e^{-j\hat{\phi}_t}$$

where $\hat{\rho}_t = \left\{ [1 - (\hat{a}^2 - \hat{b}^2) \sin^2\theta_i]^2 + 4\hat{a}^2 \hat{b}^2 \sin^4\theta_i \right\}^{1/4}$

$$\hat{\phi}_t = \frac{1}{2} \tan^{-1} \left[\frac{2\hat{a}\hat{b}\sin^2\theta_i}{1 - (\hat{a}^2 - \hat{b}^2) \sin^2\theta_i} \right]$$

Once again if γ is needed to compute K_{xz}, then we can simply replace K_{xz} by its first approximation, k, in order to obtain a first approximation for γ.

APPENDIX B

EVALUATION OF AN INTEGRAL

The integral to be evaluated in this appendix is the one given by equation (127), which is repeated below:

$$< E_{HH}\ E^*_{HH} > = \frac{(k_o \cos\theta_i)^2 A_o}{4\pi^2 R_o^2} \int_o^{\infty} dR \int_o^{\pi} dx \int_o^{2\pi} d\phi\ R^2 \sin x\, e^{-R/\ell}\, e^{-2jk_o R \sin x \cos\phi \sin\theta_i} \cdot$$

$$e^{jqR\cos x} \Big\{ A_{y1} A^*_{y1} c_1\, e^{-jR\cos x(k_z' + k_z'^*)/2} + A_{y1} A^*_{y2} c_2\, e^{-jR\cos x(k_z' - k_z'^*)/2}$$

$$+ A_{y2} A^*_{y1} c_3\, e^{jR\cos x(k_z' - k_z'^*)/2} + A_{y2} A^*_{y2} c_4\, e^{jR\cos x(k_z' + k_z'^*)/2} \Big\}$$

The integral in ϕ can be performed directly by using the following result.

$$\int_o^{2\pi} e^{-jx\cos t}\, dt = 2\pi J_o(x)$$

The expression for $< E_{HH}\ E^*_{HH} >$ then becomes

$$< E_{HH}\ E^*_{HH} > = \frac{(k_o \cos\theta_i)^2 A_o}{4\pi^2 R_o^2} \int_o^{\pi} dx \int_o^{\infty} dR\ R^2 \sin x\, e^{-R/\ell}\, 2\pi\, e^{jqR\cos x} \cdot$$

$$J_o(2k_o R \sin x \sin\theta_i) \Big\{ A_{y1} A^*_{y1} c_1\, e^{-jR\cos x(k_z' + k_z'^*)/2} + A_{y1} A^*_{y2} c_2\, e^{-jR\cos x(k_z' + k_z'^*)/2}$$

$$+ A^*_{y1} A_{y2} c_3\, e^{jR\cos x(k_z' - k_z'^*)/2} + A_{y2} A^*_{y2} c_4\, e^{jR\cos x(k_z' + k_z'^*)/2} \Big\}$$

We will break up the above integral into four separate integrals, the first of which we will call K_1.

$$K_1 = 2\pi A_{y1} A_{y1}^* c_1 \int_o^\infty dR \int_o^\pi dx \sin x \, J_o(2k_o R \sin x \sin\theta_i) \, e^{jqR\cos x} \, e^{-jR\cos x(k_z' + k_z'^*)/2} R^2 e^{-R/\ell}$$

let $u = \cos x \qquad \sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - u^2}$

$du = -\sin x dx$

$$K_1 = 2\pi A_{y1} A_{y1}^* c_1 \int_o^\infty dR \int_{-1}^1 du \, J_o(2k_o R \sin\theta_i \sqrt{1 - u^2}) e^{ju[qR - R(k_z' + k_z'^*)/2]} R^2 e^{-R/\ell}$$

$$K_1 = 2\pi A_{y1} A_{y1}^* c_1 \Big\{ \int_o^\infty dR \int_{-1}^1 du \, R^2 e^{-R/\ell} J_o(2k_o R \sin\theta_i \sqrt{1-u^2}) \cos[u(qR - R(k_z' + k_z'^*)/2)]$$

$$+ j \int_o^\infty dR \int_{-1}^1 du \, R^2 e^{-R/\ell} J_o(2k_o R \sin\theta_i \sqrt{1-u^2}) \sin[u(qR - R(k_z' + k_z'^*)/2)]$$

The second term on the right-hand side of the above equation is zero since the integral in u is from -1 to +1 over an odd function.

$$K_1 = 2\pi A_{y1} A_{y1}^* c_1 \int_o^\infty dR \int_{-1}^1 du \, R^2 e^{-R/\ell} J_o(2k_o R \sin\theta_k \sqrt{1-u^2}) \cos[u(qR - R(k_z' + k_z'^*)/2)]$$

$$K_1 = 4\pi A_{y1} A_{y1}^* c_1 \int_o^\infty dR \int_o^1 du \, R^2 e^{-R/\ell} J_o(2k_o R \sin\theta_i \sqrt{1-u^2}) \cos[u(qR - R(k_z' + k_z'^*)/2)]$$

The following integral can be used to evaluate the integral in u from 0 to 1.

$$\int_o^a J_o(b \sqrt{a^2 - x^2}) \cos(cx) \, dx = \frac{\sin[a \sqrt{b^2 + c^2}]}{\sqrt{b^2 + c^2}}$$

$$K_1 = 4\pi A_{y1} A_{y1}^* c_1 \int_o^\infty dR \, R^2 e^{-R/\ell} \frac{\sin\left[\sqrt{4k_o^2 R^2 \sin^2\theta_i + R^2 [q - (k_z' + k_z'^*)/2]^2}\right]}{\sqrt{4k_o^2 R^2 \sin^2\theta_i + R^2 [q - (k_z' + k_z'^*)/2]^2}}$$

Letting $b_o = \sqrt{4k_o^2 \sin^2\theta_i + [q - (k_z' + k_z'^*)/2]^2}$

$$K_1 = 4\pi A_{y1} A_{y1}^* c_1 \int_o^\infty dR\, R\, e^{-R/\ell} \frac{\sin(b_o R)}{b_o}$$

The integral in R is easily evaluated by using the following integral:

$$\int_o^\infty x\, e^{-ax} \sin bx dx = \frac{2ab}{(a^2 + b^2)^2} \qquad a > 0$$

$$K_1 = \frac{8\pi A_{y1} A_{y1}^* c_1 \ell^3}{[1 + b_o^2 \ell^2]^2}$$

The second integral to be considered that makes up $< E_{HH} E_{HH}^* >$ will be designated K_2 and is given below.

$$K_2 = 2\pi A_{y1} A_{y2}^* c_2 \int_o^\infty dR \int_o^\pi dx\, R^2\, e^{-R/\ell} \sin x\, J_o(2k_o R \sin x \sin\theta_i)\, e^{jqR\cos x}\, e^{-jR\cos x (k_z' - k_z'^*)/2}$$

It can be seen that K_2 has exactly the same form as K_1, and therefore the result for K_2 can be written down directly.

$$K_2 = \frac{8\pi A_{y1} A_{y2}^* c_2 \ell^3}{[1 + b_1^2 \ell^2]^2}$$

where $b_1 = \sqrt{4k_o^2 \sin^2\theta_i + [q - (k_z' - k_z'^*)/2]^2}$

The third integral that makes up part of $< E_{HH} E_{HH}^* >$ will be given the symbol K_3.

$$K_3 = 2\pi A_{y1}^* A_{y2} c_3 \int_o^\infty dR \int_o^\pi dx\, R^2 \sin x e^{-R/\ell} J_o(2k_o R \sin x \sin\theta_i)\, e^{jqR\cos x}\, e^{jR\cos x (k_z' - k_z'^*)/2}$$

It can be seen that K_3 has exactly the same form as K_1, and the result for K_3 follows from K_1.

$$K_3 = \frac{8\pi A^*_{y1} A_{y2} c_3 \ell^3}{[1 + b_2^2 \ell^2]^2}$$

where $b_2 = \sqrt{4k_o^2 \sin^2\theta_i + [q + (k_z' - k_z'^*)/2]^2}$

The last integral that makes up part of $< E_{HH} E^*_{HH} >$ will be designated K_4 and is given below.

$$K_4 = 2\pi A_{y2} A^*_{y2} c_4 \int_o^\infty dR \int_o^\pi dx\, R^2 \sin x\, e^{-R/\ell} J_o(2k_o R \sin x \sin\theta_i)\, e^{jqR\cos x}\, e^{jR\cos x (k_z' + k_z'^*)/2}$$

Again, it is easily seen that K_4 has the same form as K_1 and so the result can simply be written as

$$K_4 = \frac{8\pi A_{y2} A^*_{y2} c_4 \ell^3}{[1 + b_3^2 \ell^2]^2} \quad \text{where } b_3 = \sqrt{4k_o^2 \sin^2\theta_i + [q + (k_z' + k_z'^*)/2]^2}$$

The final result for $< E_{HH} E^*_{HH} >$ can now be written.

$$< E_{HH} E^*_{HH} > = \frac{2k_o^2 \ell^3 A_o \cos^2\theta_i}{\pi R_o^2} \left\{ \frac{A_{y1} A^*_{y1} c_1}{[1 + b_o^2 \ell^2]^2} + \frac{A_{y1} A^*_{y2} c_2}{[1 + b_1^2 \ell^2]^2} \right.$$

$$\left. + \frac{A_{y2} A^*_{y1} c_3}{[1 + b_2^2 \ell^2]^2} + \frac{A_{y2} A^*_{y2} c_4}{[1 + b_3^2 \ell^2]^2} \right\}$$

APPENDIX C

COMPUTER PROGRAM LISTING FOR THE CALCULATION OF THE RADAR BACKSCATTER COEFFICIENTS

```
      PROGRAM RANDOM(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     RADAR SCATTERING FROM RANDOM MEDIA
C     USING A VECTOR RENORMALIZATION FORMULATION
      COMPLEX QJ,QK,QCOTI,QA,QL1,QL2,QL3,QL4A,QL4,QL5,QL,QK1,QT,QKZP,Q1,
     1Q2,Q3,Q4,Q5,Q6,Q7,Q8,QAY1N,QAY1D,QAY1,Q9,Q10,Q11,Q12,Q13,Q14,Q15,
     2QAY2N,QAY2D,QAY2,QKZPC,QAY1C,QAY2C,QC1,QC2,QC3,QC4,
     3QB0,QB1S,QB1,QB2S,QB2,Q33,QHH1,QHH2,QHH3,QHH4,QHH,QVV1,
     4QVV2,QVV3,QVV4,QVV,QLLK,QALPH,QGAM,
     5QXX1,QXX2,QXX31,QXX32,QXX3,QXX41,QXX42,QXX4,QXX5,QXX6,QXX,QEXX,
     6QXZ1,QXZ2,QXZ3,QXZ4,QXZ5,QXZ,QEXZ,QZZ1,QZZ2,QZZ3,QZZ4,QZZ5,QZZ6,
     7QZZ,QEZZ,QXD,QXN,QX,QK2,QCOSC,QTN,QTD,QM0,QN0,QF1,QF2,QF3,QF0,Q26,
     8Q27,Q28,Q29,QAX1,Q30,Q31,Q32,Q33,Q34,QAX2,Q35,Q36,Q37,Q38,Q39,Q40,
     9Q41,Q42,Q43,QAZ1,QAZ2,QAV1,QAV2,QAV1C,QAV2C
      N=3
      DO 15 I=1,N
      READ(5,101)AER,SIG,XK0L,EPS
  101 FORMAT(4F10.8)
      PI=3.14159265
      F=9.0E+9
      WRITE(6,250)F,AER,SIG,XK0L,EPS
  250 FORMAT(1H ,2HF=,E11.5,5X,4HAER=,F10.8,5X,4HSIG=,F10.6,5X,3HKL=,F10
     1.8,5X,4HEPS=,F10.8)
      E0=8.854E-12
      U=4.0E-7
      U0=PI*U
      W=2.*PI*F
      B0=W*SQRT(U0*E0*AER)
      AL0=SIG*SQRT(U0*E0*AER)/(2.*E0*AER)
      XLAM=3.*10**(8)/F
      XK0=W*SQRT(U0*E0)
      XL=XK0L/XK0
      QJ=(0.0,1.0)
      QK=B0-QJ*AL0
      R1N=B0**2+AL0*(2.*XL*B0**2+2.*XL*AL0**2+AL0)
      R1=R1N/(B0**2+AL0**2)
      R2=2.*B0*XL
      R0=SQRT(R1**2+R2**2)
      PHI0=ATAN2(R2,R1)
      QCOTI=0.5*(PHI0-QJ*ALOG(R0))
      A1=REAL(QCOTI)
      A2=AIMAG(QCOTI)
      QA=1./XL+QJ*QK
      A3=REAL(QA)
      A4=AIMAG(QA)
      QL1=1./(QK**2+QA**2)
      A5=REAL(QL1)
      A6=AIMAG(QL1)
      QL2=-QJ*QCOTI/QK**2
      A7=REAL(QL2)
      A8=AIMAG(QL2)
      QL3=-3.*QJ*(QA/QK-(1.+(QA/QK)**2)*QCOTI)/(2.*QK*QK)
      A9=REAL(QL3)
      A10=AIMAG(QL3)
      QL4A=2./3.+(QA/QK)**2-(QA/QK)*QCOTI
      A11=REAL(QL4A)
      A12=AIMAG(QL4A)
      QL4=(QL4A-QCOTI*(QA/QK)**3)/(2.*QK*QK)
```

```
      QL5=(QCOTI*QA/QK-1.)/(QK*QK)
      QL=QL1+QL2+QL3+QL4+QL5
      QLLK=1.+QL*(XK0*XK0*EPS/QK)**2
      XB=REAL(QLLK)
      YB=AIMAG(QLLK)
      PHIB=ATAN2(YB,XB)
      RHOB=SQRT(XB**2+YB**2)
      QK1=QK*SQRT(RHOB)*CEXP(QJ*PHIB/2.)
      B1=REAL(QK1)
      AL1=-AIMAG(QK1)
      AA=XK0*B1/(B1**2+AL1**2)
      BB=XK0*AL1/(B1**2+AL1**2)
      THETA=0.0
      WRITE(6,111)
  111 FORMAT(2X,5HTHETA,14X,1HP,10X,2HP1,8X,5HSIGHH,11X,5HSIGVV.10X,3HXK
     1I,10X,5HSIGCM)
  102 T=PI*THETA/180.
      SN=SIN(T)
      CN=COS(T)
      Y1=2.*AA*BB*SN*SN
      X1=1.+(BB**2-AA**2)*SN*SN
      RHO1=(X1**2+Y1**2)**0.25
      PHI1=0.5*ATAN2(Y1,X1)
      P=RHO1*(B1*SIN(PHI1)+AL1*COS(PHI1))
      Q=RHO1*(B1*COS(PHI1)-AL1*SIN(PHI1))
      QT=2.*XK0*CN/(Q-QJ*P+XK0*CN)
      XKX=XK0*SN
      XKY=0.0
      XKZ=SQRT(XK0**2-XKX**2-XKY**2)
      Y2=-2.*AL0*B0
      X2=B0**2-AL0**2-XKX**2-XKY**2
      RHOZ=SQRT(Y2**2+X2**2)
      PHIZ=ATAN2(Y2,X2)
      QKZP=SQRT(RHOZ)*CEXP(QJ*PHIZ/2.)
      QKZPC=CONJG(QKZP)
      XKR=SQRT(RHOZ)*COS(PHIZ/2.)
      XKI=SQRT(RHOZ)*SIN(PHIZ/2.)
      Q1=QKZP*XKZ**2+QKZP*XKX**2+XKZ*QKZP**2+XKZ*XKX**2
      Q2=XKZ*(XKZ+QKZP)*EPS*QT*(XK0*XKX*XKY)**2/(2.*QJ*QKZP)
      Q3=Q1*XKZ*QKZP*XK0*XK0*EPS*QT/(2.*QJ)
      Q4=XKZ*Q1*(XKY**2+QKZP**2)*XK0*XK0*EPS*QT/(2.*QJ*QKZP)
      Q5=-QKZP*(QKZP+XKZ)*(XKX*XKY)**2
      Q6=XKZ*(XKY**2+QKZP**2)*Q1
      Q7=-XKZ*(XKZ+QKZP)*(XKX*XKY)**2
      Q8=Q1*(QKZP*XKY**2+QKZP*XKZ**2)
      QAY1N=Q2+Q3-Q4
      QAY1D=Q5+Q6+Q7+Q8
      QAY1=QAY1N/QAY1D
      Q9=XKZ*Q1*(XKY**2+QKZP**2)*XK0*XK0*EPS*QT/(2.*QJ*QKZP)
      Q10=-XKZ*(QKZP+XKZ)*EPS*QT*(XKX*XKY*XK0)**2/(2.*QJ*QKZP)
      Q11=Q1*XKZ*QKZP*XK0*XK0*EPS*QT/(2.*QJ)
      Q12=-QKZP*(QKZP+XKZ)*(XKX*XKY)**2
      Q13=XKZ*(XKY**2+QKZP**2)*Q1
      Q14=-XKZ*(XKZ+QKZP)*(XKX*XKY)**2
      Q15=QKZP*(XKY**2+XKZ**2)*Q1
      QAY2N=Q9+Q10+Q11
      QAY2D=Q12+Q13+Q14+Q15
      QAY2=QAY2N/QAY2D
```

```
QAY1C=CONJG(QAY1)
QAY2C=CONJG(QAY2)
QC1=1./(2.*P-QJ*QKZP+QJ*QKZPC)
QC2=1./(2.*P-QJ*QKZP-QJ*QKZPC)
QC3=1./(2.*P+QJ*QKZP+QJ*QKZPC)
QC4=1./(2.*P+QJ*QKZP-QJ*QKZPC)
QB0=(2.*XK0*SN)**2+(Q-(QKZP+QKZPC)/2.)**2
BB0S=REAL(QB0)
BB0=SQRT(BB0S)
QB1S=(2.*XK0*SN)**2+(Q-(QKZP-QKZPC)/2.)**2
X1B=REAL(QB1S)
Y1B=AIMAG(QB1S)
RHOB1=SQRT(X1B**2+Y1B**2)
PHIB1=ATAN2(Y1B,X1B)
QBB1=SQRT(RHOB1)*CEXP(QJ*PHIB1/2.)
QB2S=(2.*XK0*SN)**2+(Q+(QKZP-QKZPC)/2.)**2
X2B=REAL(QB2S)
Y2B=AIMAG(QB2S)
RHOB2=SQRT(X2B**2+Y2B**2)
PHIB2=ATAN2(Y2B,X2B)
QBB2=SQRT(RHOB2)*CEXP(QJ*PHIB2/2.)
QB3=(2.*XK0*SN)**2+(Q+(QKZP+QKZPC)/2.)**2
B3=REAL(QB3)
BB3=SQRT(B3)
QHH1=QAY1*QAY1C*QC1/(1.+(BB0*XL)**2)**2
QHH2=QAY1*QAY2C*QC2/(1.+(QBB1*XL)**2)**2
QHH3=QAY1C*QAY2*QC3/(1.+(QBB2*XL)**2)**2
QHH4=QAY2*QAY2C*QC4/(1.+(BB3*XL)**2)**2
QHH=QHH1+QHH2+QHH3+QHH4
HH=REAL(QHH)
SHH=8.*(XL**3)*(XK0*CN)**2*(HH)
DBSHH=10.*ALOG10(SHH)
QALPH=XK0*SN/(B0-QJ*AL0)
AH1=B0*XK0/(B0**2+AL0**2)
BH1=AL0*XK0/(B0**2+AL0**2)
Y11=2.*AH1*BH1*SN**2
X11=1.-(AH1**2-BH1**2)*SN**2
RH021=(X11**2+Y11**2)**0.25
PHI21=0.5*ATAN2(Y11,X11)
QGAM=-RH021*CEXP(-QJ*PHI21)
QXX1=QGAM**2/(QK**2+QA**2)
QXX2=QJ*(3.*QALPH**2-1.)*QCOTI/(QK*QK)
QXX31=QJ*3.*(2.*QALPH**2-QGAM**2)/(2.*QK*QK)
QXX32=QA/QK-(1.+(QA/QK)**2)*QCOTI
QXX3=QXX31*QXX32
QXX41=(2.*QALPH**2-QGAM**2)/(QK*QK)
QXX42=1.-(QA/QK)*QCOTI
QXX4=QXX41*QXX42
QXX5=QL4*3.*QGAM**2
QXX6=-2.*QL4
QXX=QXX1+QXX2+QXX3+QXX4+QXX5+QXX6
QEXX=(QK/XK0)**2+QXX*(XK0*EPS)**2
QXZ1=3.*QJ*QGAM*QALPH*QCOTI/(QK*QK)
QXZ2=-3.*QGAM*QALPH*QL4
QXZ3=-QGAM*QALPH*QL1
QXZ4=9.*QJ*QGAM*QALPH*QXX32/(2.*QK*QK)
QXZ5=3.*QGAM*QALPH*QXX42/(QK*QK)
QXZ=QXZ1+QXZ2+QXZ3+QXZ4+QXZ5
```

```
QEXZ=QXZ*(XK0*EPS)**2
QZZ1=QL1*(QALPH)**2
QZZ2=QJ*(3.*QGAM**2-1.)*QCOTI/(QK*QK)
QZZ3=QJ*3.*(2.*QGAM**2-QALPH**2)*QXX32/(2.*QK*QK)
QZZ4=(2.*QGAM**2-QALPH**2)*QXX42/(QK*QK)
QZZ5=3.*QALPH*QALPH*QL4
QZZ6=-2.*QL4
QZZ=QZZ1+QZZ2+QZZ3+QZZ4+QZZ5+QZZ6
QEZZ=(QK/XK0)**2+QZZ*(XK0*EPS)**2
QXD=QEXX*QALPH**2+QEZZ*QGAM**2+2.*QGAM*QALPH*QEXZ
QXN=QEXX*QEZZ-QEXZ**2
QX=QXN/QXD
X=REAL(QX)
Y=AIMAG(QX)
RHO=SQRT(X**2+Y**2)
PHI=ATAN2(Y,X)
QK2=XK0*SQRT(RHO)*CEXP(QJ*PHI/2.)
QALPH=XK0*SN/QK2
BETA2=REAL(QK2)
ALPH2=-AIMAG(QK2)
AH=BETA2*XK0/(BETA2**2+ALPH2**2)
BH=ALPH2*XK0/(BETA2**2+ALPH2**2)
RHO2X=1.-(AH**2-BH**2)*SN**2
RHO2Y=2.*AH*BH*SN*SN
RHO2=(RHO2X**2+RHO2Y**2)**0.25
PHI2=0.5*ATAN2(RHO2Y,RHO2X)
QCOSC=RHO2*CEXP(-QJ*PHI2)
QGAM=-QCOSC
QTN=2.*QK2*XK0*CN
QTD=QK2*XKD*CN+QCOSC*XK0**2
QT=QTN/QTD
P1=RHO2*(ALPH2*COS(PHI2)+BETA2*SIN(PHI2))
Q=RHO2*(BETA2*COS(PHI2)-ALPH2*SIN(PHI2))
QM0=EPS*W*U0*QT*RHO2*CEXP(-QJ*PHI2)*XK0*XK0/QK2
QN0=XK0**3*EPS*W*U0*QT*SN/(QK2**2)
QF1=(QKZP+XKZ)*XKY**2+QKZP*XKZ**2+XKZ*QKZP**2
QF2=XKX*XKY*(QKZP+XKZ)
QF3=XKZ*(XKZ*QKZP+QKZP**2)+(XKZ+QKZP)*XKX**2
QF0=QF1*QF3-QF2**2
Q26=XKX*XKZ*QN0*QF1/(2.*QJ)
Q27=XKX**2*XKZ*QM0*QF1/(2.*QJ*QKZP)
Q28=-XKY*XKZ*QN0*QF2/(2.*QJ)
Q29=-XKY*XKX*XKZ*QM0*QF27(2.*QJ*QKZP)
QAX1=(Q26+Q27+Q28+Q29)/QF0
Q30=-XKX*XKZ*QN0*QF1/(2.*QJ)
Q31=QJ*QM0*XKZ*QKZP*QF1
Q32=-XKX**2*XKZ*QM0*QF1/(2.*QJ*QKZP)
Q33=XKY*XKZ*QN0*QF2/(2.*QJ)
Q34=XKY*XKX*XKZ*QM0*QF2/(2.*QJ*QKZP)
QAX2=(Q30+Q31+Q32+Q33+Q34)/QF0
Q35=XKY*XKZ*QN0*QF3/(2.*QJ)
Q36=XKY*XKX*XKZ*QM0*QF3/(2.*QJ*QKZP)
Q37=-XKX*XKZ*QN0*QF2/(2.*QJ)
Q38=-XKX**2*XKZ*QM0*QF2/(2.*QJ*QKZP)
QAY1=(Q35+Q36+Q37+Q38)/QF0
Q39=-XKY*XKZ*QN0*QF3/(2.*QJ)
Q40=-XKY*XKX*XKZ*QM0*QF3/(2.*QJ*QKZP)
Q41=XKX*XKZ*QN0*QF2/(2.*QJ)
```

```
      Q42=-QJ*QM0*XKZ*QKZP*QF2
      Q43=XKX**2*XKZ*QM0*QF2/(2.*QJ*QKZP)
      QAY2=(Q39+Q40+Q41+Q42+Q43)/QF0
      QAZ1=XKX*QAX1/XKZ+XKY*QAY1/XKZ
      QAZ2=XKX*QAX2/XKZ+XKY*QAY2/XKZ
      QAV1=-(QAX1*CN+QAZ1*SN)
      QAV2=-(QAX2*CN+QAZ2*SN)
      QAV1C=CONJG(QAV1)
      QAV2C=CONJG(QAV2)
      QC1=1./(2.*P1-QJ*QKZP+QJ*QKZPC)
      QC2=1./(2.*P1-QJ*QKZP-QJ*QKZPC)
      QC3=1./(2.*P1+QJ*QKZP+QJ*QKZPC)
      QC4=1./(2.*P1+QJ*QKZP-QJ*QKZPC)
      QB0=(2.*XK0*SN)**2+(Q-(QKZP+QKZPC)/2.)**2
      BB0S=REAL(QB0)
      BB0=SQRT(BB0S)
      QB1S=(2.*XK0*SN)**2+(Q-(QKZP-QKZPC)/2.)**2
      X1B=REAL(QB1S)
      Y1B=AIMAG(QB1S)
      RHOB1=SQRT(X1B**2+Y1B**2)
      PHIB1=ATAN2(Y1B,X1B)
      QBB1=SQRT(RHOB1)*CEXP(QJ*PHIB1/2.)
      QB2S=(2.*XK0*SN)**2+(Q+(QKZP-QKZPC)/2.)**2
      X2B=REAL(QB2S)
      Y2B=AIMAG(QB2S)
      RHOB2=SQRT(X2B**2+Y2B**2)
      PHIB2=ATAN2(Y2B,X2B)
      QBB2=SQRT(RHOB2)*CEXP(QJ*PHIB2/2.)
      QB3=(2.*XK0*SN)**2+(Q+(QKZP+QKZPC)/2.)**2
      B3=REAL(QB3)
      BB3=SQRT(B3)
      QVV1=QAV1*QAV1C*QC1/(1.+(BB0*XL)**2)**2
      QVV2=QAV1*QAV2C*QC2/(1.+(QBB1*XL)**2)**2
      QVV3=QAV1C*QAV2*QC3/(1.+(QBB2*XL)**2)**2
      QVV4=QAV2*QAV2C*QC4/(1.+(BB3*XL)**2)**2
      QVV=QVV1+QVV2+QVV3+QVV4
      VV=REAL(QVV)
      SVV=8.*(XL**3)*(H*E0*CN)**2*(VV)
      DBSVV=10.*ALOG10(SVV)
      NK=THETA
      IF(NK)15,260,261
  260 SIGCO=SHH
      DBSCM=10.*ALOG10(SIGCO)
      GO TO 16
  261 SIGCC=SIGCO*CN
      DBSCM=10.*ALOG10(SIGCC)
   16 WRITE(6,106)THETA,P,P1,DBSHH,DBSVV,XKI,DBSCM
  106 FORMAT(1H ,2X,F5.1,10X,F7.3,5X,F7.3,3X,F7.3,11X,F7.3,5X,F8.3,5X,F7
     1.3)
   14 IF(THETA-80.)107,15,15
  107 THETA=THETA+10.
      GO TO 102
   15 CONTINUE
      STOP
      END
```

AD-A047 669 ARMY ENGINEER TOPOGRAPHIC LABS FORT BELVOIR VA F/G 20/14
BACKSCATTERING OF RADAR WAVES BY VEGETATED TERRAIN.(U)
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LIST OF SYMBOLS

$\underline{r}$	position vector
$\epsilon_r(\underline{r})$	relative dielectric constant of the random medium
ϵ_a	average relative dielectric constant of the random medium
σ_a	average conductivity of the random medium
ϵ_s	standard deviation of the dielectric fluctuations
$\mu(\underline{r})$	random portion of the relative dielectric constant in the random medium
$B(\underline{r} - \underline{r}')$	autocorrelation function of $\mu(\underline{r})$
ω	radian frequency of the incident wave
$\hat{\epsilon}_a$	relative mean complex dielectric constant of a forest of leaves
V_ℓ	volume of one leaf
N_ℓ	total number of leaves in the forest volume
$\hat{\epsilon}_\ell$	relative complex dielectric constant of one leaf
V_A	volume of the forest which is solely air
ϵ_A	dielectric constant of air
V_T	total volume of the forest
ϵ_o	dielectric constant of free space
θ_i	angle of incidence
$\underline{E}$	total electric field in the random medium
$\underline{H}$	total magnetic field in the random medium
∇	del operator
μ_o	permeability of free space
$\underline{E}_s$	scattered electric field in the random medium
$\underline{\underline{\Gamma}}(\underline{r}, \underline{r}')$	infinite space dyadic Green's function
k_o	propagation constant in free space
k	mean background propagation constant in the random medium
$\underline{\underline{I}}$	unit dyadic
$<\underline{E}(\underline{r})>$	mean electric field in the random medium
$\underline{K}$	effective propagation vector of the mean wave in the random medium
$\underline{\underline{\hat{\epsilon}}}$	relative complex dielectric tensor

$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}$, etc.	elements of the relative complex dielectric tensor
$\underline{a}_x, \underline{a}_y, \underline{a}_z$	unit vectors in the x, y and z directions respectively
α, β, γ	direction cosines of the $\underline{K}$ vector
$R_\perp$	reflection coefficient (horizontal polarization)
$T_\perp$	transmission coefficient (horizontal polarization)
P	attenuation of the mean wave in the random medium (horizontal polarization)
k_x, k_y	Fourier variables
$\underline{E}_s$	scattered electric field in the upper medium (air)
σ_{HH}°	radar backscatter coefficient for the case of horizontal polarization transmit – horizontal polarization receive
$R_{\parallel}$	reflection coefficient (vertical polarization)
$T_{\parallel}$	transmission coefficient (vertical polarization)
$\hat{P}$	attenuation of the mean wave in the random medium (vertical polarization)
σ_{vv}°	radar backscatter coefficient for the case of vertical polarization transmit – vertical polarization receive

Hevenor, Richard A.
Backscattering of radar waves by vegetated terrain / by Richard A. Hevenor – Fort Belvoir, Va. : U.S. Army Engineer Topographic Laboratories : for sale by National Technical Information Service, 1977.

89 p. ; 25½ cm. (U.S. Army Engineer Topographic Labs. ; ETL 0105)

Prepared for Office, Chief of Engineers, U.S. Army.

Appendices: – A. Calculation of the elements of the dielectric tensor – B. Evaluation of an integral – C. Computer program listing for the calculation of the radar backscatter coefficients.

1. Electromagnetic waves – Scattering– Mathematical models 2. Radar
I. Title II. (Series)